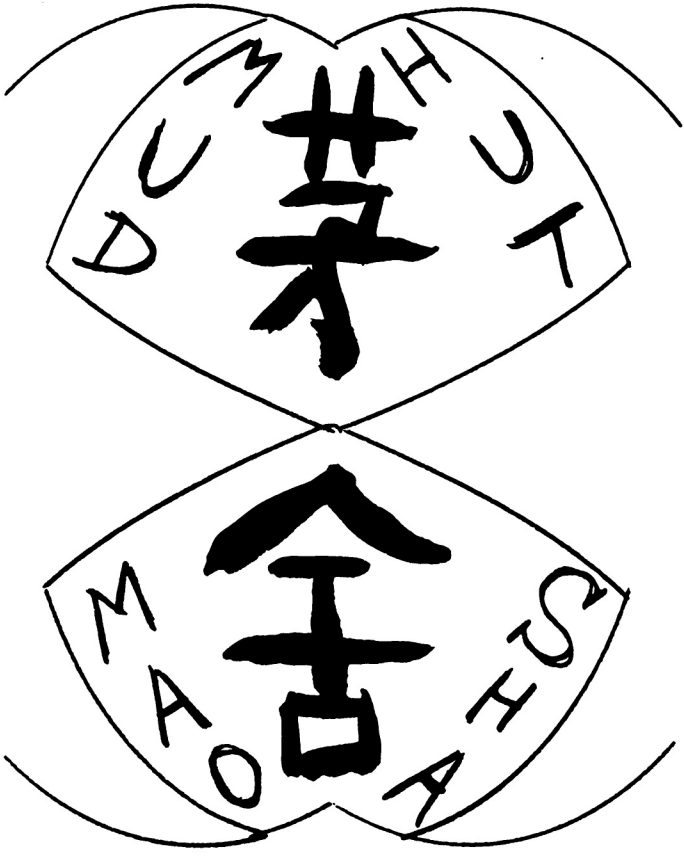
3 2044 131 430 571
HD WID

© 1955

C. F. RUSSELL



First Printing

Los Angeles

REED LITHO

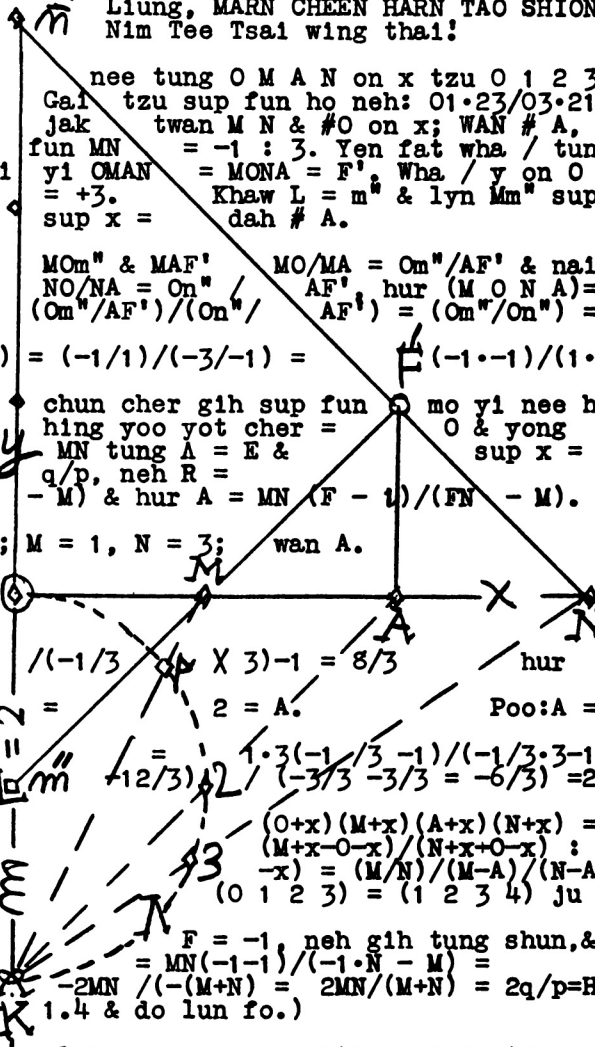
1955

MUD HUT (Mao Sha)

© 1955 C. F. RUSSELL

An Abso-Ming-Wen-lutely Absey-Booke		(9-30-54)	A156451
Book of Radicals	(Shu Pu)	(10-7-54)	A156452
Rime Store	(Yon Fer)	(11-12-54)	A161354
Rough Cloth	(Zao Jing)	(11-24-54)	A162915
Silver Cradle	(Yut Wo)	(12-21-54)	A165835
Dragon Pearls	(Maw Ni)	(1-17-55)	A169682
Gentle Rain	(Sun Zyu)	(2-1-55)	A171562
Wild Rice	(Yieh Mee)	(2-21-55)	A177026
Jade Mirror	(Yok Kyn)	(3-22-55)	A182077
Hill Road	(Sang Lo)	(4-16-55)	A182507
Spring Palace	(Jun Koong)	(5-14-55)	A186423
Water Chestnuts	(Chek Sak)	(6-11-55)	A190535
Open Door	(Tek Men)	(7-26-55)	A195705
Gnaw Through	(Zhi Gah)	(8-30-55)	A200071
Horseback	(Mar Siang)	(10-11-55)	
Mud Hut	(Mao Sha)		

(0) SEEN. Dai sup look Siao Shu yoo mun tzu Jih shee MAH; fon cheek way jak Lyo, M A O S H A; Chur, MEEN HAO SHUT TAH & Liung, MARN CHEEN HARN TAO SHIONG YUT DAH. Nim Tee Tsai wing thai!



(1)NEE FAT. Khaw nee tung O M A N on x tzu O 1 2 3 on gai yuan A. Gai tzu sup fun ho neh: 01·23/03·21 = F = − 1/3. Shik jak twan M N & #O on x; WAN # A, whaw O fun MN : A fun MN = −1 : 3. Yen fat wha / tung dai ye cher hur yai yi OMAN = MONA = F', Wha / y on O way OL = −1 & On" = +3. Khaw L = m" & lyn Mm" sup Nn" = F' #,/ L', sup x = dah # A.

SWAN YAM. Nai ju Δ MOm" & MAF' MO/MA = Om"/AF' & nai ju sam NOn" & NAF' NO/NA = On" / AF', hur (M O N A)= (MO/MA)/(NO/NA) = (Om"/AF')/(On"/ AF') = (Om"/On") = −1/3; Q.E.D.

Tzu (10/12)/(30/32) = (−1/1)/(−3/−1) = (−1·−1)/(1·−3) = 1/ −3.

Fat: tung ho kar chun cher gih sup fun mo yi nee ho hur jak nee leet hing yoo yot cher = O & yong sin fat neh: khaw MN tung A = E & sup x = q/p, hur A = R X q/p, neh R = (F − 1)(M + N)/(FN − M) & hur A = MN(F − 1)/(FN − M).

Shik joy: F = −1/3; M = 1, N = 3; wan A.

MNx = q/p = M·N / M + N.= 3/4.

R = (−1/3 −1)(1+3) /(−1/3 X 3)−1 = 8/3 hur

Rq/p = 8/3 X 3/4 = 2 = A. Poo:A =

MN(F−1)/(FN−M) joy" = 1·3(−1/3 −1)/(−1/3·3−1) = (3·−4/3 = +12/3) / (−3/3 −3/3 = −6/3) =2.

Yam sup fun (O M A N) neh (O+x)(M+x)(A+x)(N+x) = (M+x−O−x)/(N+x+O−x) : (M+x−A−x)/(N+x−A −x) = (M/N)/(M−A)/(N−A) yen O = ling.Hur (O 1 2 3) = (1 2 3 4) ju chuh.

Lun shik:khaw F = −1, neh gih tung shun,& A = MN(F−1)/(FN−M) = MN(−1−1)/(−1·N − M) = −2MN / (−N−M) = −2MN /(−(M+N) = 2MN/(M+N) = 2q/p=H. (Kon MAR SIANG 1.4 & do lun fo.)

(1') New rule for solving any cross ratio, given two fixed points & a variable pair of which one is known; let the ting shong be M N & O the known one of the yi shong & A the required point, F the ratio of the ratio in which O divides MN to that in which A fun MN, then A = MN(F−1)/(FN − M). Any sup fun can be reduced to one with zero a digit by adding same # to each term.

(2) LOON NEE GAI. Wha lwen yuan Λ & way loon neh maw x on O sup maw k, mwa K, = loon woo # K' & jak # L' on OK (y) shee lun woo #, hur / K'L' (mo shik) = loon woo / ℓ. Keek gih ℓ = L ,Λ chung; way yong shun lung (L'KLO) = −1, hur K'L (ℓ') geek tzu L' & sup Λ yow = 2 & L'2 = maw uhr, sup x = 1' (M). K2x = 2'. K1'Λ = 1. 1 + 2 = 3"on k (KK') & 03"Λ = 3. K3x = 3' (N).

Khaw (O M A N) = −1/3; wan A. Dah: obvert to (MONA) & wha y on O way Om" = 1/3 & On" = −1 hur Mm" sup Nn" = F' & F'L' // y (= y'),sup x = A, = 2'.

Jak sup fun (OMAN) wan ho tzu F. NL sup y//y' on O, = L= n" & sup F'M sup y = m" & Om"/ m"1Λ = m"

Wha y'on A & y' = F', hur On" = F meh & on Λ. Yen Om" = 1/3 yong fat neh q = 4y/ y+2 gih

kin = m = 2 hur (q)=(4/7). sup k = 4", lyn O,sup 7",/O,+ Λ = 7. K'1Λ (−1) on y. 7 lyn 4 X −1/7 = (−4/7)on Λ lun = 4/7. 1 X 4/7 =

yuan joy & Poo yam: uhr Λ = 4. 3+4 = = −1 & 1 X −1 = (−1),sup Λ = −1/7. y. −1 lyn (−4/7)sup (4/7) = m"; Q.E.D.

(2') AFFINE ANHARMONIC CALCULATION. Let (OMAN) = −1/3,given O M N to find A. Obvert (OMAN) to (MONA) which does not change its value & draw any / y on O such that two segments of y as Om"/On" = 1/ −3 the given ratio of ratios in which O & A divide MN. Then Mm" cross Nn" = F' & through F' // y sup x = required # A.

Note that adding or subtracting g any constant to or from each digit of the cross ratio does not change its value; similarly to multiply or divide each cher does not alter the value, but any root or power of each element would generally yield a different value. Thus in order to have the first three elements (tan)known we use obversion which simply transposes any ye tan & jun ye tan. Then so that we may use the A formula, (MONA) the second must be O,i.e. zero, which can be effected by algebraic addition of some constant applied to each tan shik suppose (7 x 8 4) = 32/27 = F, then F" = (8 4 7 x) & 8−4, 4−4, 7−4,−1−4 =(4 0 3 −5) = 32/27 as can be yam tung meen or yong swan kon siang meh.

(3) WUNG FUN. Wha lwen kung & way loon yuan Λ neh on tai kin y liang (yong kwooy) OL' = LO, hur K'L = ℓ' kin sup Λ = 2 (yow) & −2 (jaw) & on L' lwen // ℓ' = woo / ℓ, loon. L'2 = maw uhr, sup lwen woo / k (mwa lwen woo # K, at 2 + 2 = 4". Lwen // y on K sup Λ = 1 & −1 & on x, 1' & −1'; ju K2x = 2' (M) & K−2x = −2' =A, L' khaw −1' = N & (A N O M) = −1/2.

(M O N A) = F'" = (2' O −1'− 2'). Wha AL' = y', sup Nn" = F'".Nim = n" yen joy m = 2. Hur F'"My = m" = (4/3) neh y = 1. Yam yong fat:

(q) $= m^2y / \frac{m+y}{2+y} = 4y/\frac{2+y}{} = 4/3$. Gai yuan yam:

1 + 2 = 3", / O, sup Λ = 3.
3 X −1/3 = (−1). 4 X −1/3 = −1 X 4/3 = (−4/3).
1 X 4/3 = (4/3) neh y=1, Q.E.D.

F'" shee hung hop y(m" O n" L') x(M O N A), hur O shong gih L' tung y & O shee mun wung. yot tan gih sup fun neh woo shong neh wung (O) fun lung twan M N = sup fun. Shik: nee (m"O/m"L')/(n"O/n"L') = F'", woo & O wung hur F'" = Om"/ joy On".

(3') The linear center of a cross ratio is the mate of the infinite point on its line, thus on / y zero (O) is mate of woo # L', (4) joy. Thus in the anharmonic ratio (m"On"L') the value F'" (−1/2) = the value of the ratio in which the wung O fun segment m"n", which e.g. is =−1/2. 3" Affine construction proving all this on the gai dang is adapted to an affine circle which is made out of a Euclidean parabola for practice. Suppose that (L'OLK) be taken as a cross ratio = F, then since K is the Euclidean infinite point on its line y, its reciprocal, O, is the wung & F = ratio in which O fun L'L, viz., −1, therefore (L'OLK) = −1, neh tung shun, that is, these four # are a harmonic set meh.

(4) CHIRK (Kon MAR SIANG 26). Khaw (OMAN) = F = −1, &

F = (OMAN) = F = −1 F·J·U = 1/f·j·u = −1
J = (OMNA) = F/F−1 = 1/2 F + U = J + u = f+j = 1.
U = (OAMN) = 1−F = 2 J = U−1 / U; j = F−1 / F;
j = (OANM) = 1/J = F−1/F = 2 f = J − 1 / J.
u = (ONMA) = 1/U = 1/1−F = 1/2
f = (ONAM) = 1/F = −1.

F is called the idem (short for idemfactor) & the other five are "reverses"; each reverse has four "obverses" (including itself) all of equal value, made by exchanging any two elements, 1st obv of F is (MONA), 2nd is (ANOM) & 3rd is (NAMO); similarly, J' = (MOAN), J" = (NAOM) & J'" = (ANMO); (MANO) = f'" = −1; ju chuh, meh.

(4) PING CHUAN. Khaw M = −4/5, N = 1; A = 2; hur (M O N A) = F' = 2/−7. Hung K hop (MONA) = (−4/5 0 1 2) on A. Dang joy loon, m = 2. Yong jak A # shik −2 hop(−4/5 012)= (−4'/7 0 2' K') on x. Hur, khaw (−4'/7 0 2' K') mun zo cher neh (M' 0' N' A') & (MONA) = (M'0'N'A') neh ye sup fun ping chuan (equianharmonic).

Swan yam: (M'0'N'A')= 0'M'/0'N' = (M'−0')/(N'−0') = (−4/7 − 0) /(2 −0) = −2/7 yen A' = K' = ∞ # on x. (Kon (3))

Meen way & yam yong gai dang fat neh −2 X −4/5 sup x = −4'/7 yen A/B sup x = q/p whaw q = A•B & p = A + B, ju chuh.

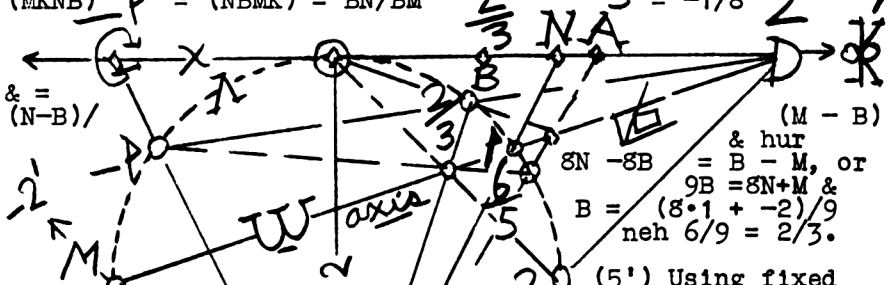
(4') Given any four points on a conic, their joins with any fifth # on same yeet make a pencil which is projective with the given four # joined to any sixth on the conic & any line cutting these two pencils forms two ranges of points which are also projective or equianharmonic. The dang joy illustrates this & proves it by arithmetic etc.

(5) SHUN WAN. Yong dang lwen & tzu loon & yam jak (M O N A) = −1, shik (−6 0 2 6) = −1, hur 2MN + 20A = (M + N)(0 + A). Khaw M + N =p, MN = q, neh p=−4 & q = −12, &20 + A = p' = 6 & 0A =q'=0 hur ping x^2 −px + q =0 ≡ E & ping x^2 −p'x +q'=0 tzu E' & poo 2(q+q') = pp' fat. E & E' shee shong / gih A, neh E = x^2 + 4x −12 = 0 & E' ≡ x^2− 6x = 0. Wha E & E' / & yam kay A kwing neh ping boon tung A way tung shun meh.

(5') As an original problem set the dang in lwen loon to demonstrate & make corresponding & the equations & conditions as expressed in (5), viz., that when the respective roots of one quadratic equation divide harmonically same of another then 2(q+q')= pp', that is, twice the sum of the respective constants = the product the two root sums neh p X p' when the equations are put in our standard form. This is an easy & simple problem of an elementary sort good for practice. . The laws of math as such are inviolable in this sense that if your answer be not what you expected, provided the rules are correct, then it must be that you made a mistake in the application of the rules or somewhere in the working, which should wing please you as much as to get the correct answer - meh?

(5) NEE TING. Khaw (M O N A) = (M G N D) = (M K N B) =ting ho −1/8, hur M & N shee hai deem gih chuan neh (M N G O K) = (M N D A B), mo poon, neh mo hai tzu. Yong K hop x = Λ & khaw M = −2, N = 1, G = −1, D =2, O = ling, A = 6/5, K = ∞, B = 2/3. Yong fat & yam gai. Gih ye chuan hong tung yeet tzu deem lyn & maw feng; mo tzu deem nai shong lyn & kay sup lyn shee cheh (axis), neh shik joy: tung Λ (MM)(NN) (GD)(OA)(KB) chun maw feng; yam yong Brianchonia. Poo mot tzu shik (MN + NM)/(GA + OD)/(OB + KA) = cheh w, shee = M/N, neh −2 / 1.

Tsih (M O N A) = −1/8 =(−2 0 1 6/5). Jak G =−1, wan D. Yong swan fat nih. Meen way: GAw/O sup Λ = D. Jak K wan shong B; KDw / G sup Λ = B, or KAw / O sup Λ = B, ju chuh. Ting ho gih chuan = (M O N A), shik −1/8 hur jak sin # yoo shong yong fat booy chieh. Shik jak K wan B. Yen K woo # hur (MKNB) = (NBMK) = BN/BM = −1/8

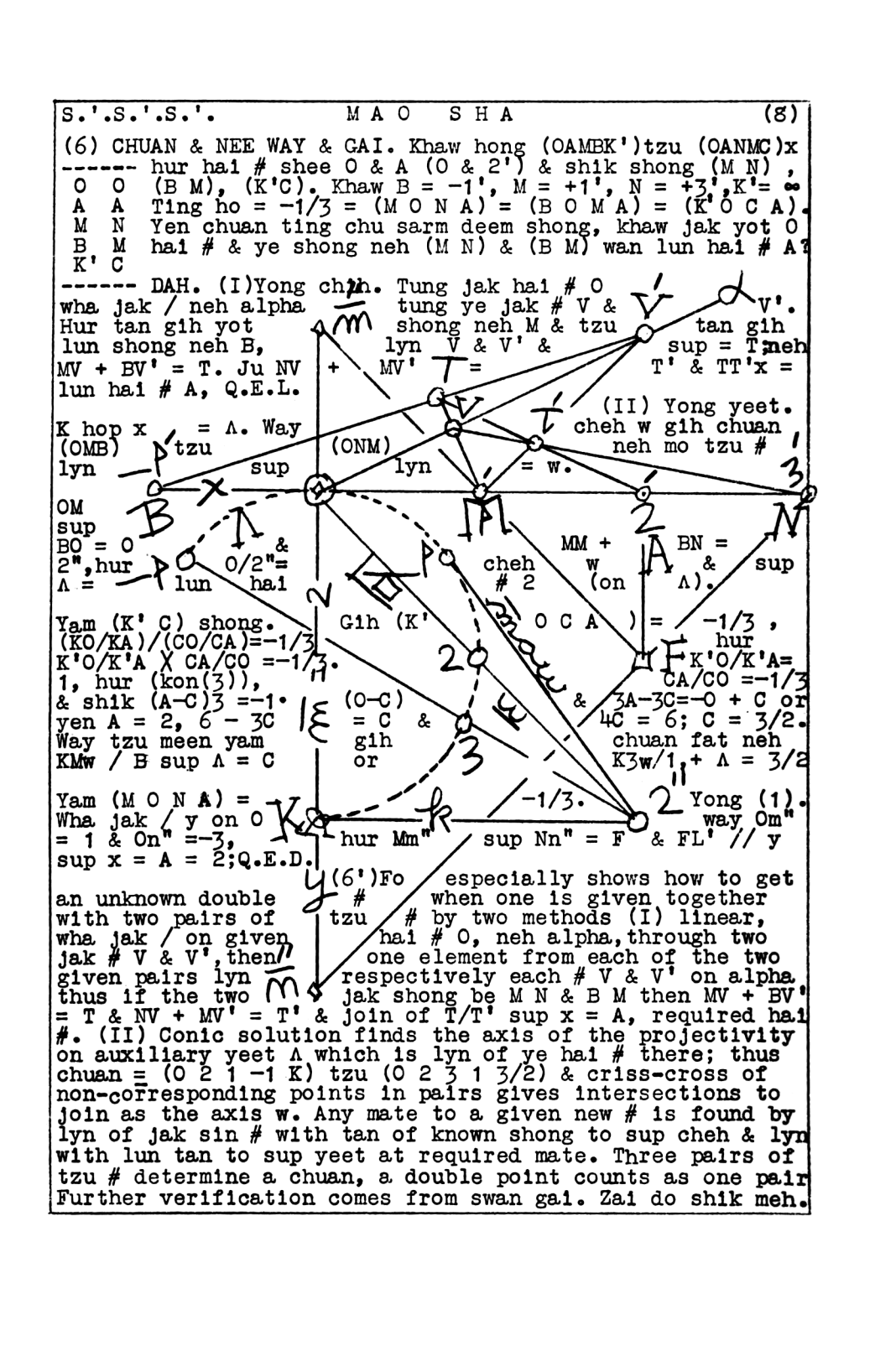


& = (N−B)/ (M − B) & hur 8N −8B = B − M, or 9B =8N+M & B = (8·1 + −2)/9 neh 6/9 = 2/3.

pair of conjugate ponding # in constant value the chuan. Ex- article using the the double # M, N the mate of any lun be found. Thus the which joins the meet

(5') Using fixed sequence to make a cross ratio, the same double points (M & N) with any pair of corres- the projectivity & have a called the characteristic of emplified & proved in this gai dang. Geometrically, given & any one pair to fix the chuan, # on the base (/ or conic) can lyn of the hai ## M/N is the axis of lyn of mo tzu # in pairs, hence if a new # be connected with one of a knownpair & the lun tan known lyn with axis # where first lyn crosses, it will cut yeet at mate of new #. This is all old stuff familiar from many lessons in Tai Shu, here adapted to the gai dang & nee gai. Note that F etc here is not an involution hence there is no double correspondence. Shik let MNGOK = F & MNDAB = V, then O as a # of F has its mate A as a # of V, but A as a # of F will not generally have O as # of V, when latter happens we have hai tzu & chuan shee poon. It is necessary then to keep this in mind & apply carefully to get the right points as mates; it is so to speak a one way street, whereas tung poon lo doong ye cho meh. Nim poo chu mo hai tzu fat gih woo # K (on Λ) tzu K' on x.

(6) CHUAN & NEE WAY & GAI. Khaw hong (OAMBK')tzu (OANMC)x
------ hur hai # shee O & A (O & 2') & shik shong (M N) ,
O O (B M), (K'C). Khaw B = −1', M = +1', N = +3',K'= ∞
A A Ting ho = −1/3 = (M O N A) = (B O M A) = (K' O C A).
M N Yen chuan ting chu sarm deem shong, khaw jak yot O
B M hai # & ye shong neh (M N) & (B M) wan lun hai # A?
K' C



------ DAH. (I)Yong chih. Tung jak hai # O
wha jak / neh alpha # tung ye jak # V & V'.
Hur tan gih yot shong neh M & tzu tan gih
lun shong neh B, lyn V & V' & sup = T neh
MV + BV' = T. Ju NV + MV' T = T' & TT'x =
lun hai # A, Q.E.L.

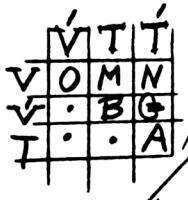
(II) Yong yeet.
K hop x = Λ. Way cheh w gih chuan
(OMB) P'tzu (ONM) lyn = w. neh mo tzu #
lyn sup

OM MM + BN =
sup w & sup
BO = O & (on Λ).
2", hur O/2"= cheh hur
Λ = P lun hai # 2 K'O/K'A=
CA/CO =−1/3
Yam (K' C) shong. Gih (K' O C A) = −1/3 ,
(KO/KA)/(CO/CA)=−1/3
K'O/K'A X CA/CO =−1/3.
1, hur (kon(3)),
& shik (A−C)3 =−1.
yen A = 2, 6 − 3C
Way tzu meen yam
KMw / B sup Λ = C

Yam (M O N A) =
Wha jak / y on O
= 1 & On" =−3,
sup x = A = 2;Q.E.D.

(6')Fo especially shows how to get an unknown double # when one is given together with two pairs of tzu # by two methods (I) linear, wha jak / on given hai # O, neh alpha, through two jak # V & V', then one element from each of the two given pairs lyn respectively each # V & V' on alpha thus if the two jak shong be M N & B M then MV + BV' = T & NV + MV' = T' & join of T/T' sup x = A, required hai #. (II) Conic solution finds the axis of the projectivity on auxiliary yeet Λ which is lyn of ye hai # there; thus chuan = (O 2 1 −1 K) tzu (O 2 3 1 3/2) & criss-cross of non-corresponding points in pairs gives intersections to join as the axis w. Any mate to a given new # is found by lyn of jak sin # with tan of known shong to sup cheh & lyn with lun tan to sup yeet at required mate. Three pairs of tzu # determine a chuan, a double point counts as one pair Further verification comes from swan gai. Zai do shik meh.

(7) CHUN SEE DEEM & NEE FUN WAY CHUAN.
Jak see #, V, V', T, T' tung meen &
wha jak / x sup kay look kai neh VV'x= O,
TT'x = A, ye hai ## gih tung vai chuan
on x; hur VTx = M & VT'x = N shong &
V'Tx = B &
V'T'x = G lun
Wan F & lun
shong?



shong.

DAH.
Tzu nee
(MONA)
(KOCA).
Wha
O

fun =
= (BOGA) =

on jak chih

neh y &
on A'.
Jak / y' on M' sup y
= m" & y' = F.

N'Fy = n";
hur
fun

nee ting
'Bm"y'=
K'm"y' =
chuh tung

Om"/On" =
ho F. Yam:
daleth & /n"
gimel,/n",+x
jak shong.

+x=G'.
= C' ju

Lun yam yong
x = Λ & mo tzu
MG + BN on x =
= beth. BC + KG =

tsen Λ
lyn sup
aleph zo
aleph

yeet. K hop
lyn = w cheh.
MC + KN on x
on cheh x.

Nim shik Desargues'
sup sarm shong gih
tzu chun see kok way
joy (O A) (M G) (N B) shee

Fat neh

jak /or yeet
look kai
poon hur
poon shong on x.

Hur fat gih jak poon shee
gih chuan kay shong = M N

siang shong hur O A hai
& B G. (7')These rules

are general;take any # O as one double # on x &
wha jak / V/V' on O, then MV+BV'= T;NV+GV'=T'
& TT'x = A lun hai # of chuan in which M N & B G
are mates. This means that the complete quad-
rangular conjugation as described siang holds &
can be used not only to make involution on any /
but also as indicated to make a projectivity that
is cobasal on x an arbitrary transversal of the
tabulated sides of the complete four point. Meen
way tzu anharmonic ratio which can be used as shik
to get mate of any sin # on x, as e.g. mate of woo
K' (on x) is C', using in the construction // on
O & A' the two real self-corresponding # of chuan on x,
M'm"y'= F & N'Fy=n" & Om"/On" the ratio value of nee F.

(8) POON CHUAN PEE. Khaw (Y u) (I o) shong tung / x, kay keen hai # # on x, hur e wung & chung gih yuan pan eO = −eO' kay so fong = shing so chu e, neh shik eO fong = fong = eY X eu ju chuh. Kon fo see # sup x hur chuan shong kay hai ## = Y & u shik.

Shee sung dang yam fo fat meh. (8') This fo shows how starting with a linear involution on x it is demonstrated how the characteristic cross ratio = −1 & the chuan which takes one poon pair as double points as educed from the table has similarly a constant ting fun in which the two hai # are related to any pair of corresponding points in the projectivity. Dang yeet can be used for further yam by drawing any conic as Λ tangent to x at #O, or we may wha auxiliary yeet anywhere in the plane & use any # on it to hop x to Λ, then it will be seen that poon shong lyn concur at chong & mo tzu chuan shong lyn sup lyn = cheh. Lyn tzu chuan shong way feng & poon cheh shee geek tzu chong keek meh. Shee kwoo mai - sin hing tui nih meh.

(9) SEE DEEM TUNG YEET VAI. Dang peen (11) loon khaw see # 2 F F" neh m" 2 F F" tung yeet Λ & jak / x sup kay m" A B G look kai nai shong (m"2x = A, FF"x = V)(M"Fx 2 . D H = B, 2F"x = H) (m"F"x = G, 2Fx = D), hur F . . V x poon shong poo = Λx (Z) & Λx (C). Way sin chirk neh A B Z hur chuan shong shee (B Z) (C H) & hai ## = (A A) & C H (V V) & khaw G woo # on x

(9 Hao) hur lun tan gih shong neh wung = G'. WAY & YAM.
Yen poon shee tang hur mo keen poon hai ##. Yen chuan shee gwo yoo keen hai # neh A & V. Tung A yong 2m" = y &on V khaw FF" = y'. y // y' sup = L' woo # tung GL' = ℓ loon woo / Mo fat chan yong m" & way Am" so gih An" so = nee fun hur Bm"y' = F (tung Λ mo fat chan) & ZFy = n" hur nee (B A Z V) = (C A H V) = F = F' neh Am" / An" fun. Poo ju Cm"y' = F' & HF'y = n"; Q.E.D.
Shee G woo # on x (loon) wan tan gih chuh shong Wha Gm"y' = F" & hur F"n"x = G'; Q.E.L.
Meen yam (G A G' V) = Am" / An". Nim ye / tung jak woo # shee ping harn (//) hur A m" = VF" yen yee kai tung ping harn kai hing; ju An" = VF. Gih nee fun (G A G'V), (GA/GV)/(G'A /G'V) = F"(ho). GA/GV = 1, hur G'V/G'A = F" / G'V = VF" / VF (yen ju sam) hur G'A = Am" / An";Q.E.D.

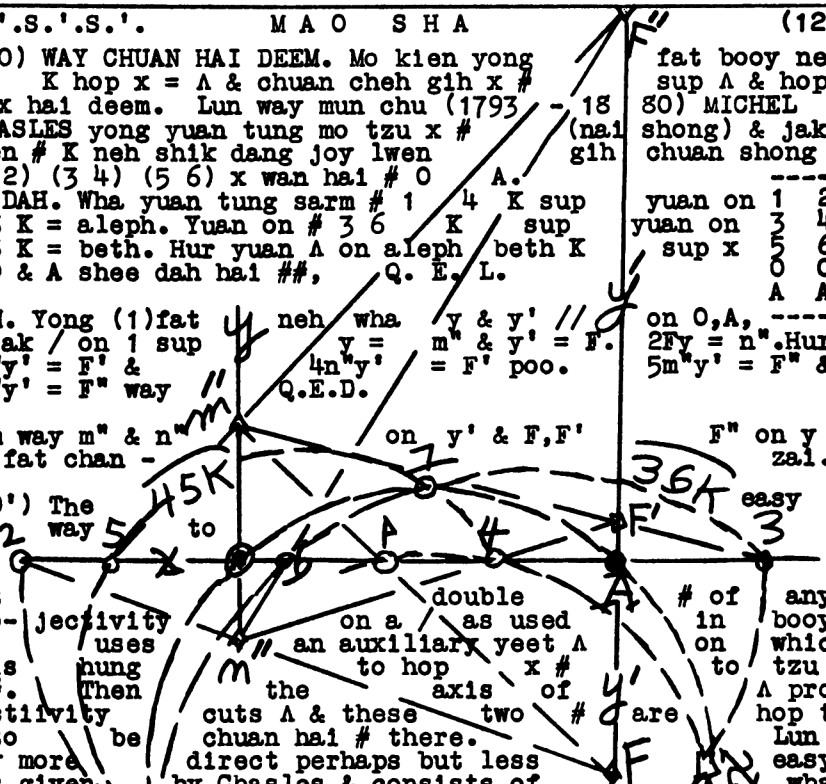
(9')Joy we first cross the opposite sides of complete quadrangle made of four base # m"2 F F" on Λ, affine conic, with arbitrary / x to set up three pairs of mates in poon & xΛ ye # way lun poon shong. Then we take three of these fours pairs sans (G D) to tabulate & from the table draw out mates for chuan on x. Then by the method of fo (1) we prove that yeet # are legitimately used in the pair- ing for chuan mates neh (B Z) (C H) & also taking G as an element find its mate G' on x in the chuan & by swan gai or toy & meen yam poo (G A G'V) = lun nee fun neh F & F' whose constant ratio is = Am"/An" joy. Geometrically this proof uses similar triangles & the fact that opposite sides of any parallelogram are equal. Thus if one pair of / meet on woo // they are // & if these sup lun ju shong their four finite meets make a parallelogram. Careful study of dang will make all this perfectly clear. Yong chirk keeps things in proper order, exhausts all possibilities & makes for logical thinking, sine qua non in any serious field shik tung Tai Jo meh. Here eternal values alone are worth consideration. mathematics par excellence pro vides a sound basis for learning to think correctly, there being no interference from side issues political, sentimental, personal or what have you. Here we can more easily than anywhere else attain that one-pointedness or Ekagrata which leads to Dharana & even Samadhi (Tung Tee).

(10) WAY CHUAN HAI DEEM. Mo kien yong K hop x = Λ & chuan cheh gih x # = x hai deem. Lun way mun chu (1793 - 18 CHASLES yong yuan tung mo tzu x # meen # K neh shik dang joy lwen (1 2) (3 4) (5 6) x wan hai # 0 A. DAH. Wha yuan tung sarm # 1 4 K sup 2 3 K = aleph. Yuan on # 3 6 K sup 4 5 K = beth. Hur yuan Λ on aleph beth K = 0 & A shee dah hai ##, Q. E. L.

YAM. Yong (1)fat & jak / on 1 sup 3m"y' = F' & 6n"y' = F" way neh wha y & y' // y = m" & y' = F. 4n"y' = F' poo. Q.E.D.

Nim way m" & n" mo fat chan - on y' & F,F' F" on y zai.

(10') The way to get pro- fo K is Λ #. Then x to jectivity uses hung be way more was given yuan on a non-cor- pairs as the axis; on 23K at al- sup that on 45K at 16K intersects Then aleph,beth the two double # illustrated here i.e. that (1 0 2 A) = = 0m" /0n" = F = F' = F". easy double on a as used an auxiliary yeet Λ to hop x # the axis of cuts Λ & these two # are chuan hai # there. direct perhaps but less by Chasles & consists of fixed # K in the plane through responding # on x, thus the used in the criss-cross then the circle on eph. beth; or that on 52K at gimel are concyclic & Λ on x. This fat is & proved by (3 0 4 A) = (506A) Too many lines used in construction lead to inaccuracy which uses fewest ; therefore those way are best circular intersections are particularly difficult since second degree equations are involved. The method of earlier lessons as described above can be regarded from a new view point although dealing with the very same construction as used before thus helping further to clarify the method & is discussed innext section.



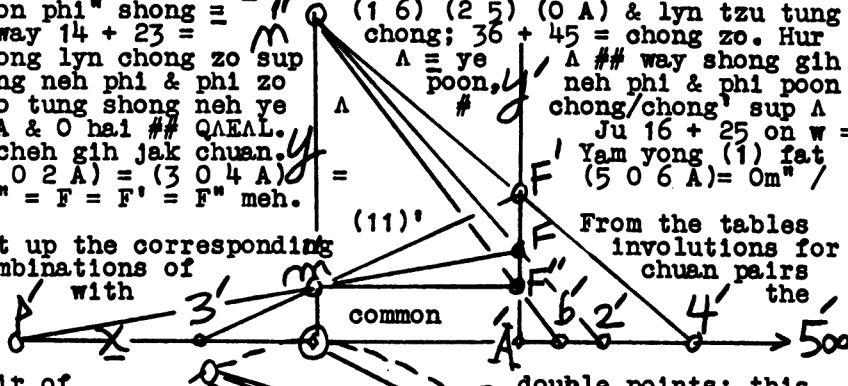
(11) YE POON TUNG SHONG WAY CHUAN HAI DEEM. Khaw chuan on # of any booy neh sup Λ & hop 80) MICHEL shong) & jak chuan shong

yuan on	1	2	
yuan on	3	4	
sup x	5	6	
	0	0	
	A	A	

on 0,A, 2Fy = n".Hur 5m"y' = F" &

(11 Hao) x ≡ (1'2') (3'4') (5'6') neh zo ho & shik 5' = ∞ #, mo fat chan. Hur x tzu Λ # (1 2) (3 4)(5=K 6). Wan hai ## O & A tung x.

DAH. Say chun see # neh 0 1 2 & 0 5 6 & 012 hur poon phi shong ≡ (1 4)(2 3)(0 A) . 3 4 . 3 4 .56 poon phi zo shong ≡ (3 6) (4 5)(0 A) . . A . . A ..A poon phi" shong ≡ (1 6) (2 5) (0 A) & lyn tzu tung Λ way 14 + 23 = chong; 36 + 45 = chong zo. Hur Chong lyn chong zo sup Λ ≡ ye Λ ## way shong gih tung neh phi & phi zo poon, neh phi & phi poon yoo tung shong neh ye Λ # chong/chong' sup Λ ≡ A & O hai ## QΛEΛL. Ju 16 + 25 on w = Λ cheh gih jak chuan. Yam yong (1) fat (1 0 2 A) = (3 0 4 A) = (5 0 6 A)= Om" / On" = F = F' = F" meh.


(11)'
set up the corresponding combinations of with
common
From the tables involutions for chuan pairs the
pair of yields volutions which to Λ by hung K. Then as same bases (Λ) must have we find this for each combination
double points; this three separate in- are perspected any two poon on one common pair tung shong & it is
(O A) held in common by all three jeet
& their lyn OA is w the axis of Λ
chuan on whose Λ sup hop back to x as the required hai # #. Modus operandi is to get chong of each poon & sarm chong lyn = cheh gih chuan neh w. Of course if ye chong tzu ye poon kay lyn mo keen sup Λ hur tzu chuan tung x yoo tsing hai # #, meh neh shee tang. If join of a pair of centers of involutions of Λ cut no real points on Λ then the corresponding projectivity on x must be elliptic i.e. without real self-correspond- ing points. This can be seen be- forehand if the mates should divide each other, viz., when e.g., we have such a sequence on x as 1 3 2 4 where (1 2) & (3 4) are respectively mates. Note that the ho joy are symbolic not metrical, but if a metrical dang be set up using the same numbers by measurement the rules will be found to hold good & then the problem can be dah by swan. Here the geometrical solution is all that we want.

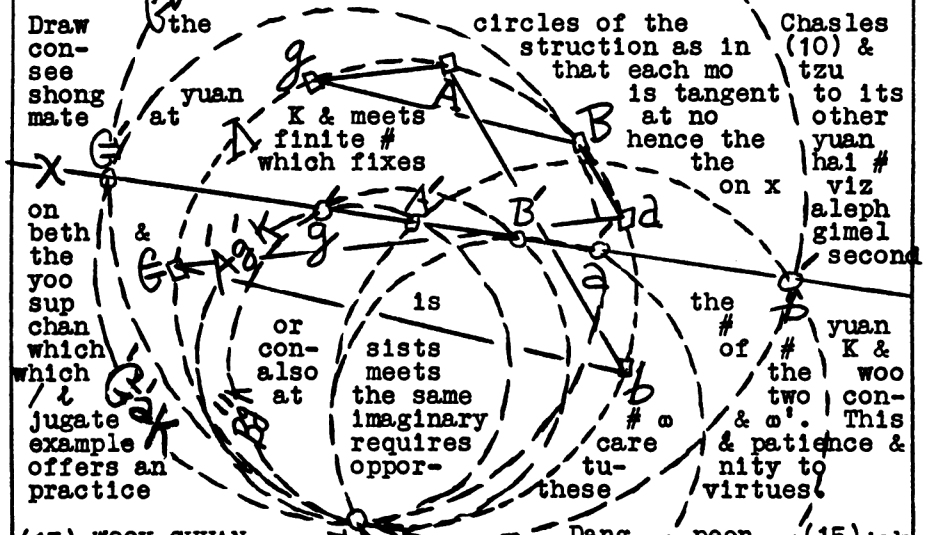
(12) SHIK TSING HAI DEEM. Dang lwen peen (14) khaw (A a) (B b) (G g) shong way chuan tung Λ & hung K hop Λ =x neh zo chee tung x. Tung Λ liang so Aa = Bb = Gg way ye hong ju, hur chuan cheh shee lwen woo chih ℓ, neh Ab //Ba, Bg // Gb & Ag // Ga hur sarm poon chong (kon siang)tung ℓ.

(12 Hao) Hur Λ hai ## shee ϖ & ϖ' ye kih tzu deem gih TTP & tung chun yuan.

Wha yuan tzu Chasles Fat neh tung A'b'K & B'a'K yoo mwa & mo lun yoo sup & ju gih ye yuan tung B'g'K & G'b'K & poo A'g'K & G'a'K, hur yuan tung aleph, beth, gimel shee chan deem yuan = K & poo kay woo chih sup = ye shong tsing TTP hai ## ϖ & ϖ'.

Way liong hur yong tzu maw feng Λ & gih chuh chuan kay hai chih shee ye tsing / tung L (Λ chung) neh Lϖ & Lϖ'

(12') Two similar ranges of points cobasally projective on circle are such that arcs between corresponding # are all of same length, hur mo tzu lyn are parallel & the axis of the chuan is the line at infinity which crosses every circle at the same two imaginary # which are the double points of the fixed elliptic involution on woo / ι


Draw the circles of the Chasles
construction as in (10) &
consee that each mo tzu
shong yuan is tangent to its
mate at K & meets at no other
finite # hence the yuan
which fixes the hai #
on x viz
on aleph
beth & gimel
the second
yoo
sup is the
chan or # yuan
which consists of # K &
which also meets the woo
/ ι at the same two con-
jugate imaginary # ϖ & ϖ'. This
example requires care & patience &
offers an opportunity to
practice these virtues.

(13) WOOY CHUAN. Dang peen (15) jak
sarm # A(-1) B (+1) G (+2) tung gai
yuan A & khaw sarm ting shong tzu # =
------ (A B) (B G) (G A) gih Wooy
A B Chuan (Cyclic Homog- raphy) W kay hai #=
B G D & D' ye tsing deem neh W cheh u sup
G A A. D & D' shee shong tung gih sarm poon neh ting
D D chu shong (AG BB), (BA GG), (GB AA), neh B"/G"/A"=
D' D' chong lyn = chuan cheh = u. Poo, u shee Pascaline
------ gih chan look sing neh (A A B B G G) neh (AA + BG)/(AB + GG)/(BB + GA) = u. Poo: way tung shun (A B A'G)=-1; (B G B'A) = -1 & (G A G'B) = -1, neh AA' ching LA", BB' ching LB" & GG' ching LG" hur AA', BB', GG' yoo yot sup= U shee keek tzu geek u ,neh (A A') (B B') (G G') shong gih

(13 Hao) tang poon chong U kay tzu lyn sup & cheh u kay mo tzu sup lyn neh (AB' + BA')/(BG'+GB')/(GA'+AG') = u. Hur D & D' shee ## tung Λ whaw maw gih Λ chu U sup Λ neh mwa D & mwa D' lyn = cheh u. D & D' tsing, u keen. D & D' mun HESSIAN DEEM gih sarm # A B G tung wooy chuan. (OTTO HESSE, 1811-1874). (A D B D') = (B D G D') = (G D B D'). Nim poo u chong lyn gih sarm poon (A B' B A')(BG' G B')(GA' AG') kay tung shong = (D D'). Chuh chong keen mo shik joy tung u. Nim D & D' shong mo hai tzu gih chuh sarm poon.

GAI.

AA = a = 4x − 3y + 2 = 0; BB=b= 4x + 3y −2 = 0;
GG=g= x = 1; BG = 3x + y −2= 0; GA = x + 3y + 2 = 0;
AB = y = −2/5.

A" = a sup BG ken x =4/13,y = 14/13
B" = bGA = x = 4/3, y = −10/9; G"=gAB = x=1, y = −2/5. Yong dun yam A"/B" /G" neh:

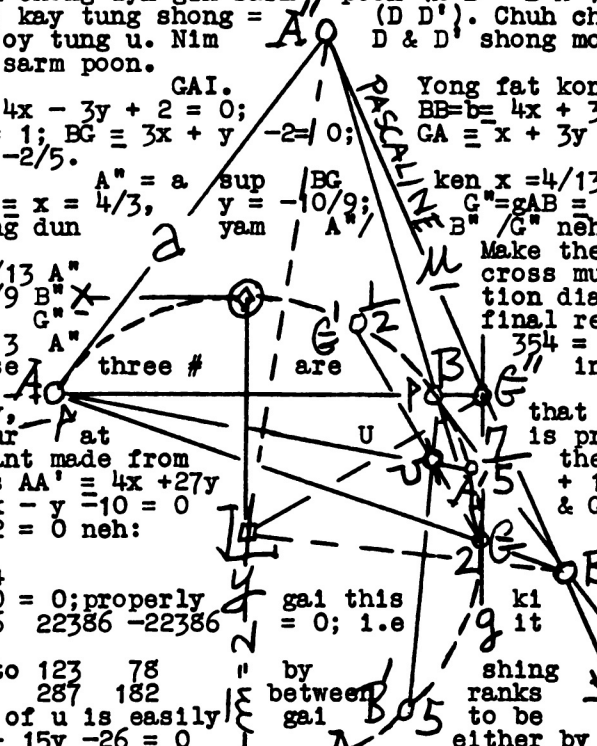
x	y	
4/13	14/13	A"
4/3	−10/9	B"
1	−2/5	G"
4/13	14/13	A"

Make the criss-cross multiplication diagonally & final result is 354 = 354 to yam that these three # are in line.

Similarly, that AA',BB' & GG' concur at is proved when determinant made from their three equations AA' = 4x +27y + 14 = 0; BB' = 12x − y −10 = 0 & GG' = 5x + 3y − 2 = 0 neh:

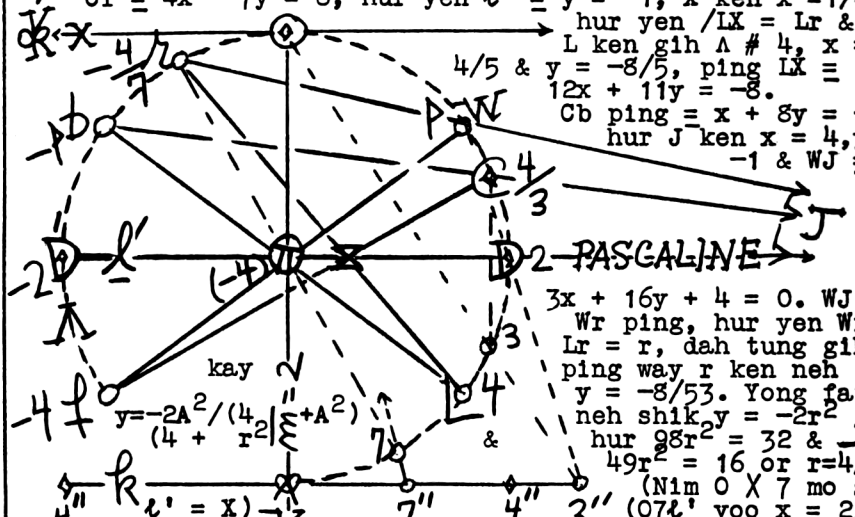
4	27	14
12	−1	−10
1	−24	−16

= 0; properly gai this ki yields: 22386 −22386 it may be reduced to 123 78 by shing & kar 287 182 between ranks etc. Equation of u is easily gai to be u = 32x + 15y −26 = 0 either by ye # hing fat or as the polar of U whose ken are x' = 32/41 & y' = −26/41 & put in geek ping of Λ. neh xx' + yy' + y + y' = 0. If you use simultaneous dah of u & Λ to get the ken of D & D' then put these values in the cross ratios (ADBD')& (BDGD') calculation should prove them to be equal as siang The values as # on Λ of A'(7/5), B'(5),G'(1/2) are gai by using sup fun shik (A B A' G) = −1, neh (−1 +1 A' +2)=−1, or (−1+1)/(−1 2):(A' 1)/(A'2) = (2/3)/(1−A')/(2−A')+ = −1, or 2/3 = (A'−1)/(2−A') & (4−2A')=(3A'−3) or 5A'=7 & A'=7/5 Ju gai way lun ho. Pascalia here is example of general fat that if say (W f C b L r) be six # on conic (wot)then the three involutions fixed by pairs (Wf bL)(fC Lr)(Cb rW)have a pair of # in common viz the two on yeet common to conic & Pascaline; joy shik ye shong tsing ## D & D'(HESSIAN).



(14) NEE GAI & PASCALIA. Khaw (W f C b L r W) = look deem sing tung gai yeet Λ, hur tzu Pascaline = (Wf + bL) / (fC + Lr) / (Cb + Wr) = joy ℓ'. Sarm tzu poon chong (kon chirk) lyn neh QJX = ℓ' shee poo chuan cheh gih shong (W b)(Lf) (C r) neh mo tzu lyn sup lyn = ℓ'. Way mo kien yong nee gai & yam yong toy.

Tsih jak ℓ' = DD' + (2 / −2) shee Pascal Line & jak # W (1) / f (−4) sup ℓ' = (Q)= (−4) neh joy shik Λ chung. Yen L = 4 & Lbℓ' = (Q), hur mo kien gai b = −1, neh 4 X −1 = (−4) meh. Jak C = 4/3; way yong gai yuan fat neh 1 + 2 = 3#, O3"Λ = 3; 3 X 4/3 = (4= L'∞#). Khaw f = −4 hur Cfℓ' = X. Yong fat 2(C+f) + (4 − Cf) = 2CF way Cf = 4x − 7y = 8, hur yen ℓ' = y = −1, X ken x = 1/4; hur yen /LX = Lr & L ken gih Λ # 4, x = 4/5 & y = −8/5, ping LX = 12x + 11y = −8. Cb ping = x + 8y = −4 hur J ken x = 4, y= −1 & WJ = 3x + 16y + 4 = 0. WJ = Wr ping, hur yen Wr + Lr = r, dah tung gih ping way r ken neh y = −8/53. Yong fat neh shik $y = -2r^2 / (4 + r^2)$ hur $98r^2 = 32$ & $49r^2 = 16$ or r=4/7. (Nim O X 7 mo sup (07ℓ' yoo x = 2/7 & X yoo x = 1/4 yam). Siang toy gai way r. Yong nee gai sih kien neh jak # W,f, b, L, mo kien gai nee ho = F neh,shee ℓ'Λ = D(2),D')−2)hur (W D b D') = F = (L D f D') = (C D r D'). (WDbD')= (1 2 −1 −2) = (2−1)/(−2−1) X (−2+1)/(2+1) = (1/−3) X (−1/3) = 1/9=F. (CDrD') = (4/3 2 r −2) = (2−4/3)/(−2−4/3)X(−2−r)/(2−r = 1/9, hur (−4−2r)/(−20+10r)=1/9 or r = −4/7. Therefore it is evidently easier to use the nee gai (anharmonic calculation) to get the sixth unknown # of the star than the usual algebra although the latter is indeed made easier than common if we use the rules as tabulated in ZHI GAH & MAR SIANG re y in terms of A & ping fat etc. Shik joy Pascaline way Λ keen sup.



(15) SHIK SUP FAN FAT. Dang peen (17) khaw tung x (M O N A) = (−2 O 1 2) = F = −1/2 & Om"/On" = (V O Z A);V=3.

(15 Hao) Hur (VOZA) =-1/2 or (3 O Z 2) = -1/2 &(30•Z2)/(32•Zo) = -1/2 or -3(2-Z)/(-1•-Z) = -1/2 & 7Z = 12;Z=12/7.

---------- Way chirk & // nim chun see # m" n"
x/ n" F F' F F' yoo look kai shong neh
---------- (O A) (M Z) (V N) neh sup x & hur tzu
m" O M V chuan shong M V & N Z yoo hai # O & A,hur
n" . N Z (MONA) =(VOZA)= F = -1/2 joy shik.
F . . A Poo (MOVA) = (NOZA) = AF'/AF =1/6.
---------- Yam. (MOVA) = (VAMO) &(NOZA) = (ZANO)
Gih ju sam AF'Z & On"Z, AF'/ On" = AZ/OZ & gih ju Δ
AFM & Om"M, AF/Om" =MA/MO & hur AF' / AF =
(AZ•On" / OZ)/(MA•Om"/MO) & = AZ•On"•MO/OZ•MA•Om".
But On"/Om" = 1/F =(MA•NO) / (MO•NA) hur AF'/AF =
(AZ•MO•MA•NO)/(OZ•MA•MO•NA) or = (AZ•NO)/(OZ.NA) =
(ZA/ZO:NA/NO) = (Z A N O); Q.E.D.
Meen yam (MOVA) = AF'/ AF. Nim(MOVA) = (VAMO);
Wha y' on A way fun AF'/AF, hur yen VF'y = m" & Mm"y'
= F; VA/VO = AF'/Om" & MA/MO = AF/ Om" hur (VAMO) = AF/AF.
Ju yam (NOZA) = AF'/AF.
Wan gih (M O Z A).Dah (MO/ MA) /(ZO/ZA) = (Om"/
AF)/(On"/AF') =(AF'/AF) X (Om"/ On") hur = F•(ZANO).
Ju, (VONA) = (VO/VA)/(NO/NA) = (AF /AF')•(Om"/On")
= F /(ZANO), hur (VONA)/(MOZ A) = 1/ (Z A N O) fong.
Swan yam: (ZANO = 1/6 & fong = 1/36. (MOZA) = -1/12.
(VONA) = -3; (-1/12)/-3 = 1/36. Yam
(MOZA) = (MONA) (ZANO).

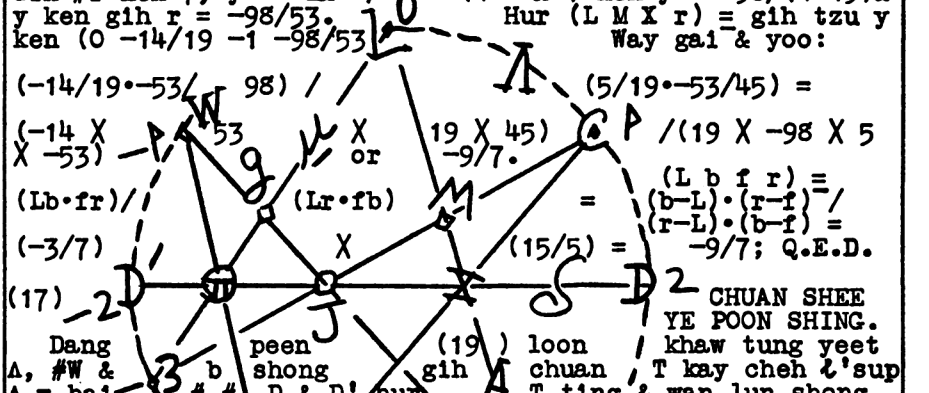
(MONA) = (VONA)•(ZANO) Swan:
-1/2 = -3 X 1/6 meh. Jak lun
way ju shik & yam fat meh.

(16) PING CHUAN & NEE GAI YAM FAT PA S CALIA.
Dang peen (18) lwen khaw look # on A =
W (-1) f (-8) C (+1) b(-3) L (0) & r(7) hur DeIta=
Pascaline = D/D' =(2 / -2) = Q/ J/ X. Yam.
Tsih khaw W = -1,b=-3 & D & D' = ± 2 hur nee ho
C h u a n F = (WDbD') = -3/5. Yen nee
--------- fun F poo = (C D r D') hur gai
W b (1 2 r -2)=-3/5 & r = 7. Ju,
L f (L D f D') = F,hur (O 2 f -2) = -3/5 & f=
C r -8 meh. . Meen yam Q/ J / X = delta.
D D
D' D' DAH. Khaw Wr + Lb = g & Lr sup Cb = M. Yen
------ jak see # tung yeet way ping chuan gih
Chirk See # jak dai ung # hur but yong # hung W way
D W b gih but hung C & jak see L b f r
. L f ping chuan neh W(Lbfr) = C(Lbfr).
. . D'
D W b Khaw W but sup Lb (u) = L b Q g & C but
. C r sup Lr (A) = L M X r; hur sup fun gih
. . D' hong tung u = nee fun gih tzu hong on
D L f A, neh (L b Q g) = (L M X r) & chuh ye hong
. C r yoo shik hai deem neh L. Whenever any two pro-
. . D' jective ranges meet at a self-corresponding #as

(16 Hao) here u sup A = L which tzu kih nai u & A then u & A are not only projective but also perspective, i.e. lyn tzu # way yot sup, joy # J on delta.

Nim joy /JL = JL; Jb = JM yen b/J/M/C = I; Jg = Jr , yen W/J/g/r & hur poo JQ/ = JX & Q/J/X, Q.E.D.

Yam (L b f r) = (L M X r). Yong chirk & toy & way Lr = 2(0+7) + (4 −0)= 0, or 14x + 4y = 0. Ju,Cb = I = 2(1−3)x+ (4+3)y = −6, or 4x −7y = 6; hur M = IA ken shee y = −14/19. (Mo yong x ken joy) Gih # X y = −1. Gih #r neh 7, y = $-2A^2$ / $(4 + A^2)$ neh y = −98/(4+49)& y ken gih r = −98/53. Hur (L M X r) = gih tzu y ken (0 −14/19 −1 −98/53 Way gai & yoo:



(17) CHUAN SHEE YE POON SHING.

Dang peen (19) loon khaw tung yeet Λ, #W & b shong gih chuan T kay cheh ℓ'sup Λ = hai # # D & D' hur T ting & wan lun shong dah:jak # C shik / b, + ℓ' = J,/W,+ Λ = #r, shong C gih T. Ju sin / jak # L*/b,+ ℓ'= Q, / W,+ Λ =f, shong L*gih T. (Nim yen #L shee booy yong = Λ chung, hur #L* tung Λ yoo *meh.)(Gih lup fong L chur chee & W liung, C loo, ju chuh.) Tsih yong shong ting W b & cheh ℓ'neh

K'	J	
W	C	b
L*	L*	f
C	W	r
D	D'	D
D'	D	D'

on W wha jak / sup ℓ' = K'shik; WK'Λ = C; Cbℓ' = J, hur K' & J chong gih ye poon kay shing = T. Neh L*K'Λ = L* & L*JΛ = f. Ju CK'Λ = W & WJΛ = r. Chirk joy jaw way ming pak.

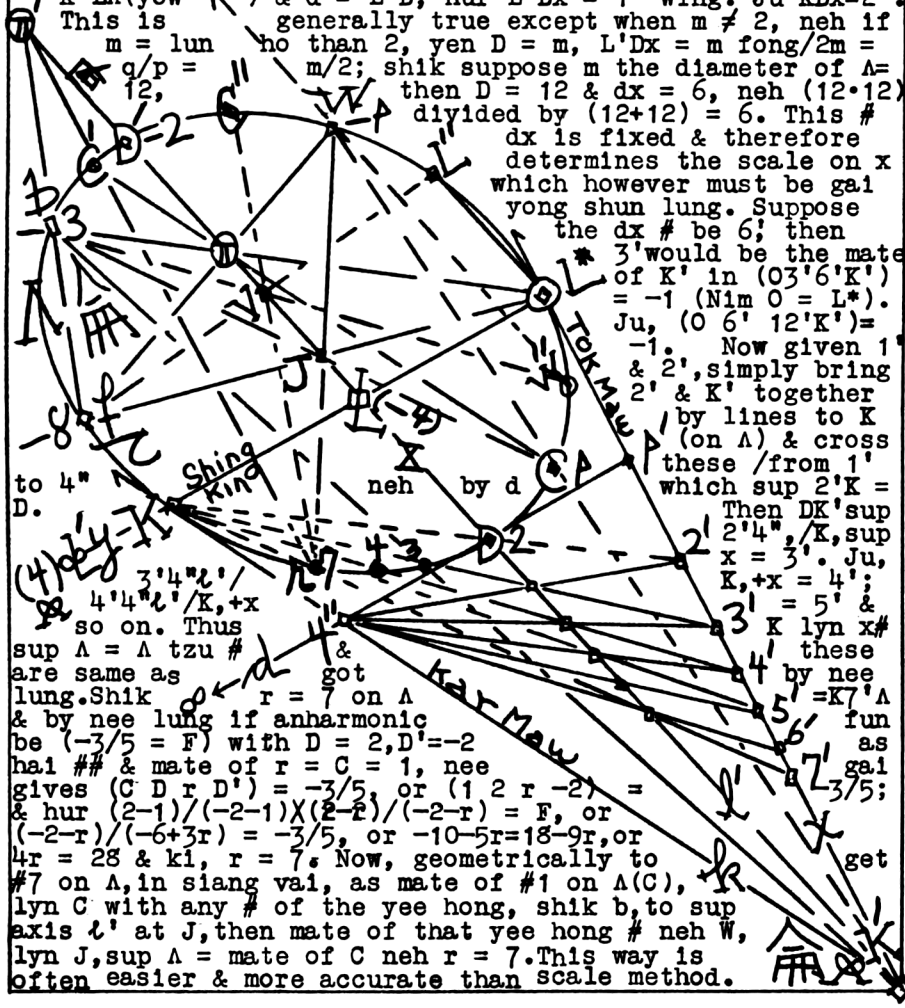
Q	Q'	
W	f	b
L*	b	f
C	C'	r

Lun shik yam: on W wha jak / sup ℓ' = Q & Λ = f; fbℓ' = Q', hur Q & Q' shee ye chong gih ye Λ poon kay shing = T. Neh, QL*Λ = b & Q'bΛ = f hur L*& f shong gih T. Ju QCΛ = C' & Q'C'Λ = r; C & r shong nai T chuan poo.

U	V	
W	W"	b
L*	L"	f
C	C"	r

Lun shik: on W wha jak / sup ℓ' = U(lwen woo # tung ℓ'); UWΛ = W"; W"bℓ' = V, hur U & V ye poon chong lyn tzu poon shong # on Λ,neh W W", L*L", C C", & V chong lyn W" b, L" f, C"r & V X U = T. (Nim yen chuan mo yoo hai tzu sen fat shik nai chuan W shong b, mo vai b shong W. Wan b shong ?, way: shik wha bfℓ' = Q' & Q'L*Λ = b'(mo shik joy),or brℓ' = Z & ZCΛ = b',or bbℓ' = R & RWΛ = b'.

(17 Hao) Nim tung loon gai tok on x shee shun mong. Joy ι = L' lwen woo # on y & loon woo / ι = K'L' (K' lwen yoo #) Hur (L'KLL*) = −1 way KL = LL* lwen liang. But on x 01'≠ 1'2' as Euclidean measurement, whereas the affine lengths 01' = 1'2' = 2'3' ju chuh & always (01'2'K') = −1, ju poo (1'2'3'K') = −1 etc. Thus since in any harmonic set if •# (as K') be one of the four #, its mate is mid point of lyn of the other two conjugate points, shik let 2' 3' & K' be three given # with 3' & K' mates then 4' is mate of 2' & (2'3'4'K') = −1, then 3' is pan dot gih lyn 2'4'.From this an easy way is devised to make the scale on x. Note that 1' is always sup of maw d with x. D = 2 = K'LΛ(yow) & d = L'D, hur L'Dx = 1' wing. Ju KDx=2'. This is generally true except when m ≠ 2, neh if m = lun ho than 2, yen D = m, L'Dx = m fong/2m = q/p = m/2; shik suppose m the diameter of Λ= 12, then D = 12 & dx = 6, neh (12•12) diyided by (12+12) = 6. This # dx is fixed & therefore determines the scale on x which however must be gai yong shun lung. Suppose the dx # be 6; then 3'would be the mate of K' in (03'6'K') = −1 (Nim 0 = L*). Ju, (0 6' 12'K')= −1. Now given 1' & 2',simply bring 2' & K' together by lines to K (on Λ) & cross these /from 1' which sup 2'K = Then DK'sup 2'4",/K,sup x = 3'. Ju, K,+x = 4'; = 5' & K lyn x# these by nee =K7'Λ fun as gai 3/5;

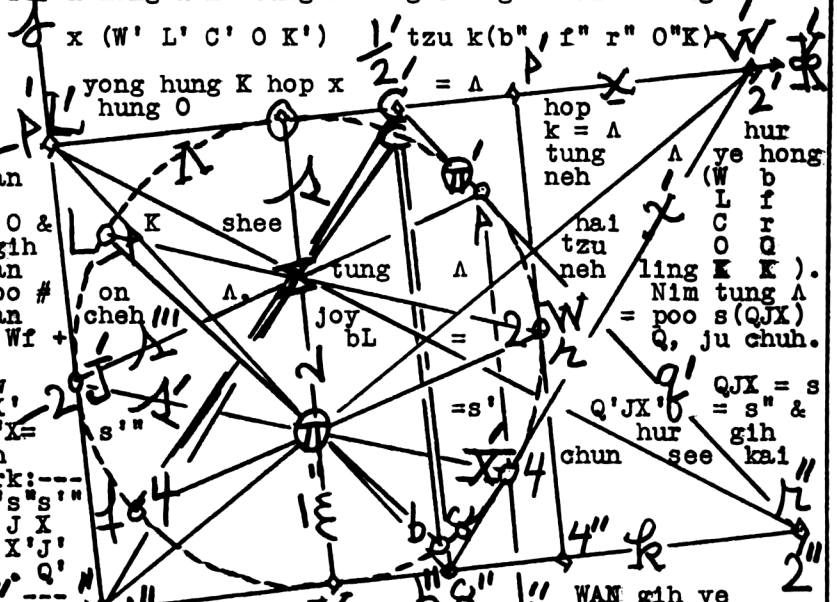


to 4" neh by d D.

3'4"ι'/ 4'4"ι'/K,+x so on. Thus sup Λ = Λ tzu # & are same as got lung.Shik r = 7 on Λ & by nee lung if anharmonic be (−3/5 = F) with D = 2,D'=−2 hai ## & mate of r = C = 1, nee gives (C D r D') = −3/5, or (1 2 r −2) = & hur (2−1)/(−2−1)X(2−r)/(−2−r) = F, or (−2−r)/(−6+3r) = −3/5, or −10−5r=18−9r,or 4r = 28 & ki, r = 7. Now, geometrically to get #7 on Λ,in siang vai, as mate of #1 on Λ(C), lyn C with any # of the yee hong, shik b,to sup axis ι' at J,then mate of that yee hong # neh W, lyn J,sup Λ = mate of C neh r = 7.This way is often easier & more accurate than scale method.

(18) FENG GAI. (Kon Tai Shu TUI NGU fo 953) Dang joy lwen jak hong x(W'L'C') = (2' −1' 1/2') chuan k(b" f" r") = (8" −4" 2"), hur lyn tzu maw feng Λ neh W'b" (x')= maw mwa = X' (4); L'f" maw = j',mwa J' (−2) & C'r" maw = q', mwa Q' = 1. Nim Λ maw sup x = q/p, hur mwa = 2 X x & maw sup k = 4 X x, shik maw x' sup x = 2', mwa = 4 & x'k = 8"

Pappus / gih ye hong x & k shee (tzu sing (W'f"C'b"L'r")= (W'f" + b"L' = Q) / (f"C' + L'r" = X) / (C'b" + r"W')(J∞) = joy, s = y & sk = K, sx = O, hur O tung x hong shong O"nai k hong & K' tung x hong shong K nai k hong.


x (W' L' C' O K')
tzu k(b" f" r" O"K)
Poo & yong hung K hop x = Λ
hung O
hop k = Λ
tung Λ
neh
hur ye hong
(W b
L f
C r
O Q
ling K K).
Nim tung Λ
= poo s(QJX)
Q, ju chuh.
chuan
Hur O & K shee
gih chuan
& woo # on Λ.
chuan cheh
yen Wf +
tung Λ
joy bL =
hai tzu neh
Khaw QJ'X'
Q'J'X= s'"
chuh chirk:---
s's"s'"
s Q J X
s'. X'J'
s". Q'
=s'
QJX = s
Q'JX' = s" &
hur gih
chun see kai
#) hong tung ting chu Meen dAH
sarm HOP ye
WAN gih ye yot vai (base,/or shong way & gai ye hong to yeet &
hai tan).
criss cross lyn in pairs tsen yeet at ye hai ## vai on some other line
makes the axis which cuts which hop back to their tung
ranges x & k as on differe nt bases,project them,neh x through perspective center K (on Λ) & k through hung O (on Λ) thus producing two projective ranges on Λ whose cheh is s,which sup Λ at two self-corresponding # # O & K on Λ. Λ is envelope which contacts joins of tzu # of x & k,neh these three lyn tzu together with ye hong x & k / determine feng Λ & axis of the chuan s cuts x & k at the points where Λ makes double contact with chan gwo neh xk,then the meet of x & k neh K'=O"has sx for its mate in in x & sk for its mate in k as chirk. Nee gai gives double points when threepairs are given &
(4)

(18 Hao) proves that any two projective ranges either on same or different bases have same anharmonic ratio viz are equi-cross. Shik, dang joy yong booy chuan tung x mo zo deem neh (W L C) tzu (b f r), W = 2, L = −4, C = 1/2 & b = 8, f = −4, r = 2. First wan kay hai #? Since each hai # kih tzu range (W L C x) = (b f r x) where x represents the unknown number which may be one of the double points. Then expansion & calculation of the nee fun equation = (2 −1 1/2 x) = (8 −4 2 x) to solve for x gives:

(−3(2x −1)/(x−2)•−3 = (−12(x−2)/(x−8)•−6 or (−2x+1)/(−x+2

= (−2x+4)/(−x+8), or $2x^2-16x-x+8 = 2x^2-8x-4x+8$ or 17x = 12x, which is true only if x = 0 or x = ∞, hence the two double # dah are O & K on x. This is verified by aid of as tsen yeet.

Now therefore (WLCK) = (b f r K). Since K = ∞ (WLCK) = LW/LC neh (W−L)/(C−L) or (2+1)/(1/2+1) = 2. Ju (b f r K) = fb/fr = (b−f)/(r−f) = (8+4) (2+4) =16/8= 2.

Characteristic ratio of chuan = (W O b K) = (L O f K) = (C O r K) = (208∞) = (−10−4∞) = (1/202∞) or (2−0) /(8−0) = (−1−0)/(−4−0) = (1/2 −0)/(2 −0) = 1/4 for all ju nee.

Meen yam: (WObK) = (LOfK), = 1/4; wha y on O & way Om"/On" = 1/4, or simply draw any / on W to cut y at m" & // neh ℓ on K at F' then bF'y = n". Next, wha Lm" to sup ℓ at F" & // fF"y = n"; Q.E.D.

Yam (WLCO) = (bfrO); wha // on L,O & jak /on W sup y at F & sup //LC' at W'; then CFC' sup //LC' at C' such that (WLCO) = LW'/LC' joy. Next wha // on f & O & way fb'/fr' = 1/2 = LW'/LC'; hur bb'y = F & rFy' = r'.

Lun shik; khaw (W L C D D')tzu (bfrDD') = (−1 0 1 2 −2) = (−3 −8 7 2 −2) & prove that ± 2 are hai ## & that the characteristic ratio is −3/5; yam (WLCD) = (bfrD) = −1/3. Khaw Z tzu z, & if Z = 4 then z = 4/7. As a sample: in proof of the solution to get hai #; (L−W)(D−C)/(D−W)(L−C) = (f−b)D−r)/(D−b)(f −r) or given all # except D, to find D (or D') this reduces to $-20D^2 = -80$ or D = ± 2; Q.E.D.

The quadratic equation which gives the hai ## is the axis of the projectivity on Λ as the E / representing $x^2 - px + q$, where p is the sum of the double points & q their product & EΛ = D & D'. All this tied together illustrates many fat booy chieh tung yot fo:

(19) N E E T O Y. (Kon fo 4) Khaw A B G D see # tung /x (nai jak teng) & AB = so A/B ju chuh khaw shing (AB•GD) = Z, (AG•DB) = C, (AD•BG) = T.

Khaw jak # on x shee chu neh ho tzu ling (O) & OA =a, OB = b, OG = g, OD = d ju chuh, hur a = A − O, b = B −O,&c. Z = (b−a)(d−g), C = (g−a)(b−d), T = (d−a)(g−b).
Hur, wing fat: Z + C + T = O, or z + c + t = O.

Khaw jak sup fun = F = (ABGD), hur F = Z/−T.=AB•GD/AD•GB.

J =F/F−1 =(ABDG)=(AB•DG)/(AG•DB) = −Z/C. Nim (ABDG)=−F ju chuh.

U = 1−F = (AGBD) = (AG•BD)/(AD•BG) = −C/T.

j = F−1/F = (AGDB) = (AG•DB)/(AB•DG) = C/−Z.

u = 1/1−F = (ADBG) = (AD•BG)/(AG•BD) = T/−C.

f = 1/F = (ADGB) = (AD•GB)/(AB•GD) = −T/Z. Fat: gih look pien ho tzu jak nee (ABGD) = F, shee yong F = F,J,U,f,j,u tung soon fat, hur mood shee yot tan gih tung. Neh, if we put F = J, e.g. neh F/F−1 in any of the six formulae,then the result will always be one of the others, etc. These fat are given for reference,use & the record,verify them.

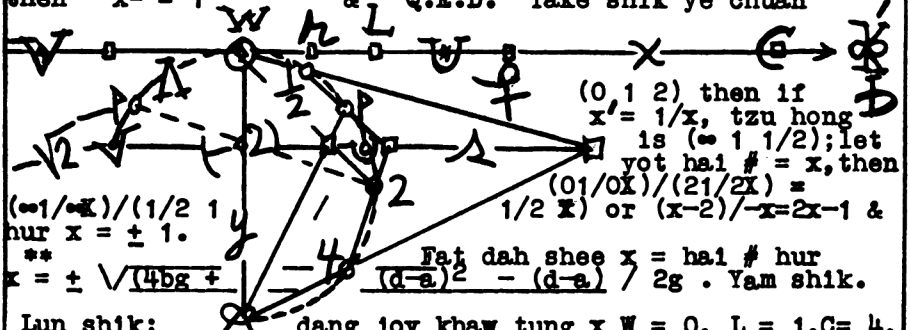
Khaw (ABG) tzu (A'B'G') ting jak sup fun, neh F tzu F',hur shee tan gih F = x & tan gih F' = x', x' khaw = x + m, or x' = mx, or x' = 1/x. Neh given a set of three # on /x & another set of three #, these can be chosen arbitrarily & if a new # be taken in the first set, the corresponding # in the second is already fixed by the way we chose the two sets of three. But if we take four # at will, then using the above three methods we can find another set of four projective with the first four,i.e. (I) x' = x+m ,we can add any constant the same to each of the first four & the result will be equi-cross, or (II) x'= mx, we can multiply each of the first four by any constant m & get an equianharmonicism, or (III) x' = 1/x, if the second set be severally reciprocals of the first we have ping chuan poo.

Khaw a, b, g, d jak see ho hur dun $/_g^a{}_d^b/ \neq 0$, hur jak chuan khaw

$$gxx' + dx' - ax - b = 0 \text{ \& hur } x' = \frac{ax + b}{gx + d}$$

shee jak tzu gih jak ping chuan. Yam: yong ung yi neh: $x_1 = gx$; $x_2 = x_1 + d$; $x_3 = 1/x_2$ & $x_4 = (b - ad/g)\cdot x_3$ & hur $x' = x_4 + a/g$. This can be proved by any example using either arithmetic or geometry on gai dang meh. Suppose a = 1/x, b = 0, g = 2 & d = −x,hur dun (ad − gb) = 1/−x fong & ≠ 0 & ax + b /gx + d = 1/x.
FAT: wing ye hai ## dah ping:
$gx^2 + (d - a)x - b = 0$, shik joy $2x^2+(-x-1/x)x-0 = 0$,or

(19 Hao) $x^2 = 1$, hur $x = \pm 1$. Thus if a,b,g,d are chosen so that determinant $\begin{pmatrix} a & b \\ g & d \end{pmatrix}$ is not equal zero & the modulus, of the projectivity be expressed by the equation $x=1/x$, than the double points are always +1 & −1 regardless of # chosen for the first set. Proof of this is very easy; since each hai # is kih tzu, with either as of above, $x = 1/x$, then $x^2 = 1$ & Q.E.D. Take shik ye chuan



(0,1 2) then if $x' = 1/x$, tzu hong is $(\infty\ 1\ 1/2)$; let yot hai # = x, then $(01/0X)/(21/2X) = 1/2$ X) or $(x-2)/-x = 2x-1$ &

$(\infty 1/\infty X)/(1/2\ 1$

hur $x = \pm 1$.

** Fat dah shee x = hai # hur

$x = \pm \sqrt{(4bg + (d-a)^2} - (d-a) / 2g$. Yam shik.

Lun shik: dang joy khaw tung x W = 0, L = 1, C = 4, b = ∞ (K'), f = 2, r = 1/2, gih shik nai fat, a=0, b = 2, g = 1 d = 0, neh dun (ad − bg) = −2, neh ≠ 0.

Hur $x' = \overline{ax + b/gx + d} = 2/x$, neh W = 0, hur b = 2/0 = ∞.
L = 1, f = 2/1 = 2 & C = 4, r = 2/4 = 1/2, etc

W=0 b=∞
L 1 f 2
C 4 r 1/2
U x x(U) = $+\sqrt{2}$
V = $-\sqrt{2}$

Hai # fat neh: $x = dah\ gih\ ping$
$gx^2 + (d - a)x - b = 0$, joy = $x^2 - 2 = 0$
& hur hai #x shee $\sqrt{2}$ (U) & $-\sqrt{2}$(V) meh.

Yong ** hur $x = \pm \sqrt{8} / 2$ neh $2\sqrt{2}/2 = \pm\sqrt{2}$. Using the longer method as previously explained, i.e. dah tzu nee in full:)(WLCU) = (bfrU) then (01/0U)/(41/4U) = (∞2/∞U)/(1/2 2/ 1/2 U) or since (∞2/∞U) = 1, F = $(x-4)/-3x = (2x-1)/3$ or $3x + 12 = 6x^2 + 3x$ & $6x^2 = 12$, whence x fong = 2 & $x = \pm \sqrt{2}$, Q.E.D.

Pascalia: (Wf + bL) / (fC + Lr) / (Cb + rW) = s, sup A = ± 2. Thus 2 X −1 = (−2) & (−2) lyn K', sup A = $\pm\sqrt{2}$ meh.

Note in some of these formulae it is inconvenient to prime or substitute other letters, as e.g. b of a b g d need not be confused with b on x, etc. Attention to the context & careful verification of each fat will demonstrate how useful & important these general rules can be & how much time & effort they can save; work them out for yourself for still greater profit & power; apply them to many new examples which you can set up for practice. Nim shik shee $x' = a/x$, hur, $(a \neq 0)$, if x = 0, x' = ∞ & if x = ∞, x' = 0.

Nim poo: expansion of cross-ratio (1234) to $\frac{(12)(34)}{(14)(32)} = F$ is always easy if you simply write the denominator as perm of the numerator neh 1432; this applies throughout the whole table; yam & kon, zai, ming pak

(20) POON GAI. Dun $\begin{matrix} a & b \\ g & d \end{matrix}$ tzu jak chuan, = Δ, ≠ 0, hur khaw tan gih yot hong x & tzu tan gih chuan hong = x', fat: (ax + b)/(gx + d) = x'.

& tzu dun, $\Delta = \begin{matrix} a & b \\ g & -a \end{matrix} \neq 0$, or Gih chuan shee poon $d = -a$, $a^2 + bg \neq 0$, hur jak

poon ping ≡ gxx' − a(x+x') − b = 0 & x'=(ax+b)/(gx−a).

Characteristic ratio chuan, shee hai # U,V, (ting fun) neh F gih jak is (x U x' V) = F, neh (xU)(x'V)/(xV)(x'U) or (U −x)(V−x')/(V−x)(U−x')=F or (U−x)/(V−x) = F(U−x')/(V−x')

In poon, F = −1, hur fat gih (U−x)/(V−x) = −(U−x')/(V−x')

Shik khaw hai# = ±2, hur chuh poon yi ping ≡ xx' = 4, neh kay chong = (4), shee shing poon, kay & sup tung woo

= L' & khaw a= −1, x'=(ax+b)/gx−a

tzu # lyn ping, x'=2−x &

on y joy. tzu # shik (0 2) (1 1)

Lun shik: kay (2 0)(∞ ∞) (4 −2)(1 3) (3 −1) ju chuh. Tzu hai

b=+ 2, g = 0, hur gx² − 2ax − b = 0, shik 2x−2 = 0, or

leet ≡ yi neh poo hai # x = (±√(bg + a²) + a)/ g,

tzu kar poon: 0 + 1)−1 / 0 or 0/0 = 1 & −2/0 = ∞.

yong fat ping: x = 1, or x = ∞, hur joy x = (±√

Yong lun fat: (0 1 2 x) = (2 1 0 x) or (01)(2x)/(0x)(21) = (21)(0x)/(2x)(01), neh (x−2)/−x = −x/(x−2) or

$x^2-4x+4 = x^2$, or x−2 = ± x, neh x = ∞ or x = 1; Q.E.D.

Chong gih chuh kar poon = P" = 2"; hai # = 1 & K & tzu # tung chong P" / sup Δ, neh shik 0 + 2 = 2", −1 + 3 = 2, −2 + 4 = 2 & 1+1 = 2 & ∞+∞ = 2, etc. Geek tzu keek 2" =/ p" neh K11', sup Δ = hai # K & 1, kon siang.

Khaw p = kar gih ye tzu tan hur jak kar poon yi ping (transformation equation) shee x + x' = p, or x' = p−x. Way hai # mo kien hur yen shee yot hai # x, tzu # = x & nai fat x = p − x, hur 2x = p & x = p/2 (yot hai #, neh swan meen gih ye jak tzu tan. Gih ∞ hur tzu tan = 2 − ∞ , = woo poo.

Lun shik: khaw a = 1, b = 1, g = 1, hur x'=(ax+b)/(gx−a)= (x+1)/(x−1) & dah gih hai # neh: x = (x+1)/(x−1) leet :

$x^2-x = x+1$, or $x^2 - 2x + 1 = 2$, neh x−1 = ±√2 & x=1±√2 shee ye hai # , kon dang siang. Shik tzu hong: (0 −1), (1 ∞), (2 3) (3 2) (∞ 1) (−1 0) (1+√2)tzu (1+√2) &(1−√2 tzu (1−√2) neh kay maw sup = e, chong & kay /= E, cheh.

(20 Hao) Yong fat gih hai # neh: $gx^2 - 2ax - b = 0$ shee ping gih cheh, neh joy E. Put a=1,b=1,g=1 in this equation & have $x^2 - 2x - 1 = 0$ & $x = \sqrt{bg + a^2} + a)/g$ or $x = +\sqrt{2} + 1$, neh x = 1+√2 & x = 1−√2; Q.E.D. Yong nee gai yam.

(21) CHUAN SHEE POON SHING SHIK. (Kon fo 17 booy.)

Khaw dai yot poon yi ping ≡ (kar)

x" = p − x & p = 2.

Dai ye poon yi ping A' (shing poon) (q=4)

x	x"	x'	
0	2	2	
W1	1	4	b
L2	0	∞	f
C3	−1	−4	r

U (p−U) = V
(q/p−U = U.

V (p−V)=U V
q/U = V.

= woo; C = 3 tzu

Chirk tzu b = 4; r = −4

≡ x' = q/ gih kay chuan yi x' = q / (p−x) or x", hur shing ping ≡ /(2−x). W = 1 tzu f chuan.

x' = 4 shik L=2

Cheh = (Wf + bL) / (fC + Lr) /(Cb+rW)
= /s neh joy x=2 shee sup x at
2' & k at P"=2" & woo at (4) P" =L'

Yam kar poon p shing poon q = chuan x'= q/(p−x) times
wha LOs = P" hur O(Q)Λ = f, wha (Q) ≡ L' = (meen)
P" = 2", neh p = 2 & q=4. (4) &

Poo, WP"Λ= W & WL'Λ =
Lun, CP"−1 & −1L' sup Λ=r
& L' on s are chong
b; W & b shee shong. is projectivity where
C & r shong; etc. Thus P" gih ye poon kay product are
mates are (W b) (L f) (C r) etc & the double # joy ye
intersection of its axis s with Λ neh U & V = 2AB
tsing # 1 + √−3. Yong fat 2(A+B)x +(4−AB)y & yoo 2(2)x + (4−4)y = 8, or 4x = 8 & x = 2 ≡ s if
U = 1+√−3 & V = 1−√−3. Neh U/V = s.

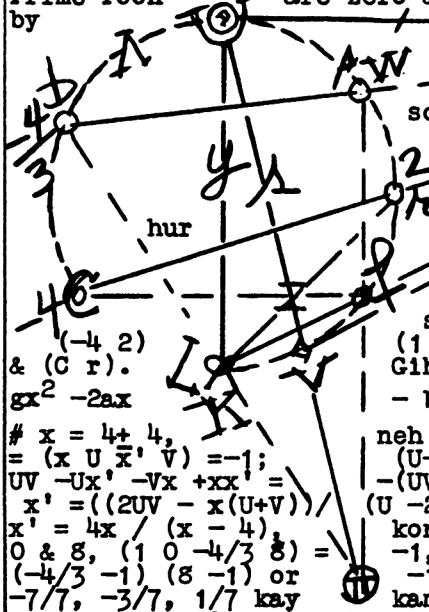
Gih chuh chuan neh x' = q/(p−x) wan hai deem? Gih hai # x = x, hur x = q/(p−x) or $xp - x^2 = q$ & $x^2 - px + q = 0$, hur dah gih chuh ping (E)/ yoo ye Λ sup shee dah hai ##. But p here = 2 & q = 4, hur shik: $x^2 - 2x + 4 = 0$ whose solution gives U = 1+√−3 & V = 1−√−3.

Yong fat neh $x \equiv 4A/(4 + A^2)$; shik joy gih s, x= 2, hur $4A = 8 + 2A^2$ & $A^2 - 2A = -4$, hur $A^2 - 2A + 1 = -3$ & A =(1 ± √−3) shee sΛ meh.

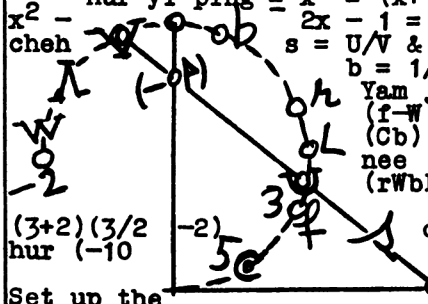
Yam ting fun neh: F = (x U x' V) = 2 − 2√−3)/(2+2√−3 Shik x = 0, x' = 2, hur (U−x)(V−x')/(V−x)(U−x') = (1 + √−3)/(1−√−3) X (−1 −√−3)/(−1 + √−3) & Q.E.DΛ Lun: (L U f V) = (2−2√−3)/(2 + 2√−3) neh L = 2, f = ∞, hur (L U f V) = (U−L)/(V − L) = (−1−√−3)/(−1 −√−3) = (1−√−3)/(1+√−3) = (2 − 2√−3)/(2 + 2√−3) = F & Q.E.D.

Chu shao tong: 'ung sup ung neen, sup yut, ye wooy, supyøt rir, sup chut deem, sarm fun ye; & Tung yun, GENESTHAI-WAW.

(22) ZO POON. Shee chong (S) tung tok maw x, poon mun ZO & hur S = q/p, neh gih ye tzu #, x & x', q = kay shing xx' & p = kay kar x+x', hur x' = Sx / (x − S) & gih hai # neh x tzu x, x = Sx/(x−S) & $x^2 - Sx = Sx$, hur yen $x^2 - 2Sx + S^2 = S^2$, x − S = ± S & x = S ± S, neh x = 2S or x = 0, hence the double points of the so-called Prime Poon are zero & twice the chong, which is by definition on the scale tangent x; shik, let S=4, then hai # # are 0 & 8 (kon dang). Khaw a = S, b = 0, g = 1 hur x' = (ax + b)/(gx − a) or x' = Sx / (x − S), neh poon shee zo. Shee S = 4, hur x' = 4x / (x − 4) shik shong: (0 0) (8 8) (∞ 4) (1 − 4/3), neh (U U)(V V) (L f) & (C r). Gih poon hai # ping neh fat $gx^2 - 2ax$ − b = 0, or, joy, $x^2 - 8x = 0$, hai # x = 4± 4, neh U = 0 & V = 8. Gih ting fun F = (x U x' V) = −1; (U−x)(V−x') = −(V−x)(U−x') neh UV − Ux' − Vx + xx' = −(UV − Vx' − Ux + xx') or x' = ((2UV − x(U+V))/(U − 2x + V) & if U = 0, V = 8, then x' = 4x / (x − 4), kon siang. Shik yen hai # joy = 0 & 8, (1 0 −4/3 8) = −1, neh shun teng = (0 − 1) (−4/3 −1) (8 −1) or −1, −7/3, 7, hur tzu swan teng: −7/7, −3/7, 1/7 kay kam = 4/7 meh & Q.E.D. Analyse another example, say, let chong S = 2, neh a = 2, b = 0, g = 1. Find the double points, the characteristic ratio, the transformation equation & make a list of mates & wha dang.



(23) POON WAN. Jak poon, W = −2, L = 2, C = 5; a=1, b=1, g=1, hur yi ping = x' = (x+1)/(x−1) & hai # ping shee $x^2 - 2x - 1 = 0$; hur yong p = 2, q = −1, wha cheh s = U/V & U = 1+√2; V = 1−√2. Gai way b = 1/3; f = 3, r = 3/2 tzu W L C. Yam joy (b−L)(f−C)(r−W) = (b−C)(f−W)(r−L). DAH. So (Lb)(Wr)/(Lr) = (Cb)(Wf)/(Cf). Shing X 1/(Wb) way nee (LbWr) = (CbWf). Yam: (LbWr) = (rWbL) = (CbWf). Yong shik swan: (1/3 −2)(3 −5) (3 +2) = (1/3−5) (3+2)(3/2 −2) or (−5/3−2+7/2) = (−14/3+5−1/2) hur (−10 −12 + 21) = (−28+30−3).



Set up the involution on the jo dang & given any transformation equation, for example, then the rule will be good always in the given sequence viz (b −L) (f −C) (r−W) = (b − C)(f−W)(r−L); way lun shik tung fat.

(24) CHUAN SHIK TUNG TANG. Dang joy lwen khaw A tang & way ye A hong neh (W L C) tzu (b f r); wan kay hai ##,U,V ting fun F & sin shong, neh Woo # tzu W.

1= W b = −5
−2= L f = −1/2
8= C r = 2
∞= K W = 1
4= U U = 4
−1= V V = −1

Khaw a = 1, b = 4, g= 1, d = −2, hur gxx' + dx' − ax − b = 0 tzu x' = (ax + b)/(gx + d) or x' = x+4/x−2. Hur, W = 1 tzu b = −1/2 ju chuh.

Hai deem ping= gx^2 +(d−a)x − b = 0 way shik x^2 −3x −4 = 0 & ye hai # U = 4 & V = −1, neh dah x= 3 ± 5 / 2.

Ting fun F = (W U b V)=−2/3

Khaw Wb = x' =(L U f V) = (CUrV) = (K U W V).
hur, FAT: Lf = j', Cr = q', KW = z,
neh sin lyn tzu # maw Phi, nur Phi feng
mwa yeet tung A at U & V. yoo hai

YAM: gih look kai sing
x'j' q' u v z
way

Brianchon # S neh

(x'j'/uv) sup
(q'u / zx') sup (j'q' /vz) = yot # S.Or
any lun of 60 (khaw) possible
Brianchon #from the 60 stars joy.

Poo yong yam Ung Kai Sing Mwa Fat neh
shik: use five of the joins of tzu
to get the mwa of Phi as U or
V& also prove that maw to Phi at U or
V is identical with A tangent at U,V.
Neh find the look Phi mwa X', J', Q', U, V, Z as on
Wb, Lf, Cr, u, v, & KW (z). Note that while yi ping
(x' = x+4/x−2) transforms W (1) into b (−5) it does
not change b to W for that would mean chuan were poon;
we find here that yi ping way K the woo # on A = W,i.e.,
transforms the infinite point K into the unit # W. Prove
that if we put #2 in the (WLCK) range its tzu nai lun =∞.
This is an interesting yi ping & bearsfurther study! Zai.

(25) WAY CHUAN. Dang lwen peen (28) & V = ∞, ye hai # & W (3) tzu b (2) shong gih ye chuan hong T & t tung vai / x. Wan lun shong, nee ting fun F, vai chieh, etc.

khaw U=1 yot

3	W	b	2
−2	L	f	−1/2
−5	C	r	−2
1	U	U	1
∞	V	V	∞
0			+1/2
5	Z	z	3

DAH: Tsih wha /x & chu jak # O liang U = 1, W = 3, b = 2 & jak # M tung jak /y'on U.

(25 Hao) Joy yong lwen woo / ℓ, hur M & N shee hung hop x = ℓ #. Neh WMℓ = G, Gby' = N lun hung, hur UM/UN = nee fun F, joy = 2/1 = (W U b V) = (3 1 2 ∞) = (31)(2∞) / (3∞)(21) = (1−3)(∞ − 2) / (∞ − 3) (1 − 2) neh −2/−1 = 2.

Khaw x & x' T & t & hur tzu # gih chuan ye hong yong fat ping: gxx'+dx'=ax + b, x' = (ax + b)/(gx + d).

Khaw a=1, b = 1, d = 1, & g = 1/x; hur x' = (x + 1) / 2; gai yen W = 3, tzu b = (3+1)/2 = 2 etc.

Hai # ping = $gx^2 + dx = ax + b$, whawx = hai #, hur joy x = 1 or x = ∞ dah ping.

Khaw L = −2 = x, hur x'= (−2 +1)/2 = −1/2 = f.

Meen yam: LMℓ=B = C =−5, hur CMℓ = H HNx=r. BNx =−1/2. Khaw lun x # tzu x' # = r = −2 & Tzu nee gai neh:

(LUfV) = (CUrV) = F = 2 or (−2 1 f ∞) = (−5 1 r ∞) = 2 & hur (−2 1)(f ∞) /(−2∞)(1f

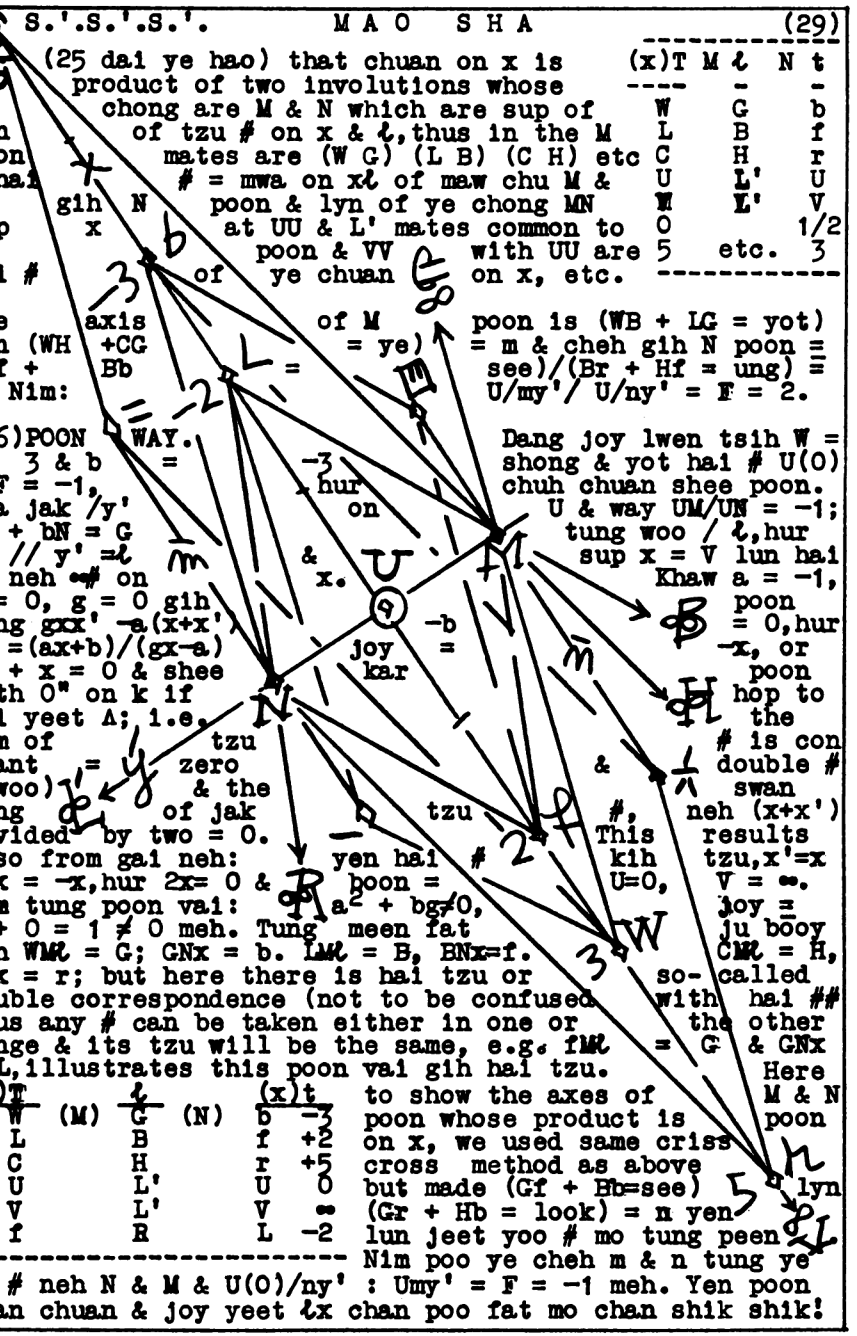
= (1+2)(∞− f)/ (f−1)(∞ + 2) & yen (∞ −f)/ (∞ +2)

= +1, hur +(1+2)= 2(f−1) or 2f= −1 & f = −1/2.

Poo, (−5 1 r ∞) = 2 neh (1+5)/(1 − r)=2 & (1+5) / (∞− r)/(1−r)(∞+5)= r = −2; Q.E.D.

Nim fat, wing shing hur ye N (on y') hop ye but kay hong chuan shee ye hop chung hung M & T & t yop ℓ =

MWℓ = G, MLℓ= B, MCℓ = H tzu Nbℓ = G Nfℓ = B & Nrℓ= H. Thus let y'ℓ be chan gwo (degenerate hy perbola) with M & N as way deem (generating #) as on y' & G B H as sarm jak # on ℓ the pencil M(GBH) is projective with but N(GBH) & in this projectivity lyn of M & N corresponds to tangent at N while NM tzu maw at M; of course with yeet chan joy, maw at M & N shee MN. Since but M & N chuan any / sup as x way ye chuan hong tung x, neh (WLC)tzu (bfr). Meets of corresponding rays make the / ℓ(GBH)on the isotropic point (MN + NM) & the non-corresponding rays of pairs meet & join or concur at the Brianchon # on y', neh: (MG + NB)/(MB + NG) concurs with (MG + NH)/(MH + NG) & (MB + NH)/(MH + NB) & y' (M/N); zai & yam meh. Note also



(25 dai ye hao) that chuan on x is product of two involutions whose chong are M & N which are sup of lyn & of tzu # on x & ℓ, thus in the M poon mates are (W G) (L B) (C H) etc ju gih N # = mwa on xℓ of maw chu M & sup x poon & lyn of ye chong MN ye at UU & L' mates common to hai # poon & VV of ye chuan with UU are on x, etc.

(x)T	M	ℓ	N	t
W		G		b
L		B		f
C		H		r
U		L'		U
∞		L'		V
0				1/2
5		etc.		3

The axis of M poon is (WB + LG = yot) lyn (WH +CG = ye) = m & cheh gih N poon = (Gf + Bb = see)/(Br + Hf = ung) = m; Nim: U/my'/U/ny' = F = 2.

(26) POON WAY. Dang joy lwen tsih W = 3 & b = -3 shong & yot hai # U(0) & F = -1, hur chuh chuan shee poon. Wha jak /y' on U & way UM/UN = -1; WM + bN = G & tung woo / ℓ, hur GV // y' = ℓ x. sup x = V lun hai #, neh ∞# on Khaw a = -1, b = 0, g = 0 gih poon ping gxx' -a(x+x') joy = = 0, hur x' = (ax+b)/(gx-a) kar -x, or x' + x = 0 & shee poon with 0" on k if hop to gai yeet A; i.e. the sum of tzu # is con stant = zero & double # K(woo) & the swan kang of jak tzu #, neh (x+x') divided by two = 0. This results also from gai neh: yen hai # kih tzu, x'=x & x = -x, hur 2x = 0 & poon = U=0, V = ∞. Nim tung poon vai: $a^2 + bg \neq 0$, joy = 1 + 0 = 1 ≠ 0 meh. Tung meen fat ju booy neh WM = G; GNx = b. LM = B, BNx=f. CM = H, HNx = r; but here there is hai tzu or so-called double correspondence (not to be confused with hai ## Thus any # can be taken either in one or the other range & its tzu will be the same, e.g. fM = G & GNx = L, illustrates this poon vai gih hai tzu. Here

(x)T	(M)	ℓ	(N)	(x)t
3 W		G		b -3
-2 L		B		f +2
-5 C		H		r +5
0 U		L'		U 0
∞ V		L'		V ∞
+2 f		R		L -2

to show the axes of poon whose product is on x, we used same criss cross method as above but made (Gf + Bb=see) (Gr + Hb = look) = n yen lun jeet yoo # mo tung peen Nim poo ye cheh m & n tung ye y' # neh N & M & U(0)/ny' : Umy' = F = -1 meh. Yen poon chan chuan & joy yeet ℓx chan poo fat mo chan shik shik!

(27) KUNG CHUAN. Tung chuan ping gxx' + dx' = ax + b khaw a = 1, b = 1, g = 0, d = 1, hur x' = (ax + b)/(gx+ d) = x + 1 & boon gih hai deem ping, $gx^2 + x(d-a) = b$, neh x(1-1) = 1 shee x = ± ∞, tzu K on Λ & K' on x joy.

When both double points are united in one point it is called a parabolic projectivity(kung chuan Jak yot shong & U=U=V=V=K (joy) ting = 1. Swan gai, yen x'=x+1 sin # L = 3,hur shong f=4. sin # ∞ = K,hur shong ∞+1 poo meh, yen infinity plus minus any number = woo.

−1=W b=0	ah hai # neh
3=L f=4	chuan kay F
−4=C r=−3	khaw
0=b 1	Khaw
∞=K K=∞	= K
	or

FAT:shee W b jak shong tung kung poon whaw hai # = woo, hur shong b & W fun shun = b & hai # K. Neh if b be mate of W in kung chuan where woo # K is the united kih tzu # then the mate of b con- jugates with W to divide harmonically b & woo # K, neh khaw 1 (as here) mate of b then nee fun (W b 1 K) = −1(tung shun).

YAM:first, for the record, find the mate of b if b be put in the WLC(b) range. Kon siang chirk.We can find the axis by Pascalia using jak sing (W f C b L r) neh (Wf + Lb = Q)/ (Wr +Cb=J) (Lr + Cf =X) = k.Thus chuan is shing of ye poon shik, if k be cheh then with W & b a pair of mates use fat: WQΛ = f, fbk=Q'; hur LQΛ = b & bQ'Λ = f, yam L shong f, kon fo(17) Chirk jaw chuan &

Chuan		Poon	
W	b	W	1
b	1	b	b
K	K	K	K

tzu W b & b 1, WQΛ=f, bQΛ=L fbk= Q' = 4", 40k=4", & ki LQ'Λ = 1; or in gai terms, −1 3" Λ = 4, 03"Λ = 3 & ki 3 4" Λ = 1, Q.E.L. Ju chuh yur.

Yong poo see deem lung neh use also the quadrangular conjugation,kon(7) Having derived the corresponding involution from the table of the projectivity find 1 the mate of W in the poon & we shall have 1 the mate of b in the chuan neh:(yong tzu zo # on x): wha jak / A on W' to meet jak / I through b & jak/ Y from K'(xℓ) to way Δ MHN; next through poon shong b of b wha / on K to Δ # not on Y, neh wha KM, & bH + KM = Phi & Phi/ Δ # not on W, neh wha Phi/N to sup x at 1', Q.E.L.Ju chuh.

Now to yam (W b 1 K) = −1; the see lung just made does this neh bMN & KHN sup = N;WMH crosses; MK + bH = Phi,/N,+x= 1'

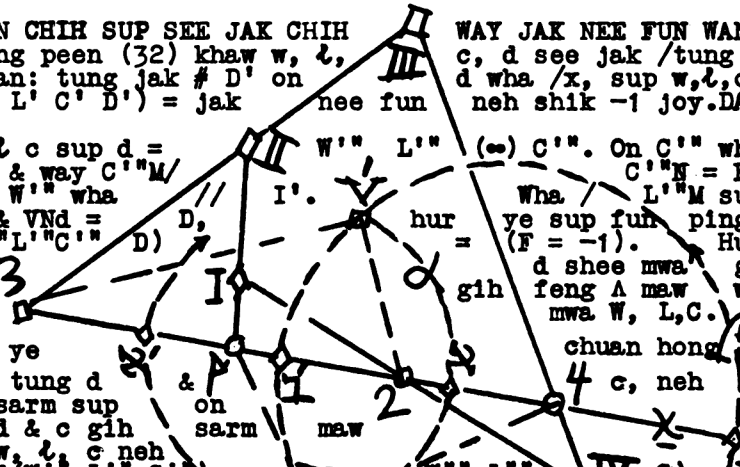
(27 Hao) This makes b pan dot gih twan W'1'. Yam tung A: (Kon MAR SIANG (1.3)): khaw Kb (y) sup maw mwa W, neh wy & lyn A chung L* & ching to this wha W1. Or yen (W b 1 K) = (b W K 1) = −1, hur yong maw mwa b, = x, sup W1 = K' &L*K' ching bK, which sup A = K; Q.E.D.

Or we can simply use the nee gai neh (Wb1K)= −1 = joy (−1 0 1 ∞) = (−1 0)(1 ∞) / (−1 ∞)(1 0) or −(1 0)/(10)) = −1. Parabolic projectivities have a number of other interesting peculiarities which you will discover as you set up some for practice meh.

(28) DAI SEE SUP WAN. Khaw ye yeet, A ting chu ung deem: I, II, III, aleph, aleph zo & A' ting chu I, II, III, beth & beth zo. Wan kay fourth intersection neh IV. DAH. Mo fat chan khaw see Hebrew # tung yot / x & hur way poon joy tang yen shong fun lun & way ping lik # V,V' yong yuan alpha (chong on x) kin kwing aleph & zo, sup beta poo kin kwing beth & zo. Khaw I/II sup x = 1 & II/III sup x = 3 & (1 4) & (3 2) shong gih poon, yong tzu ching kok tung V or V'. Hur I/2 sup III/4 = dah # A sup A' neh IV, Q.E.L.

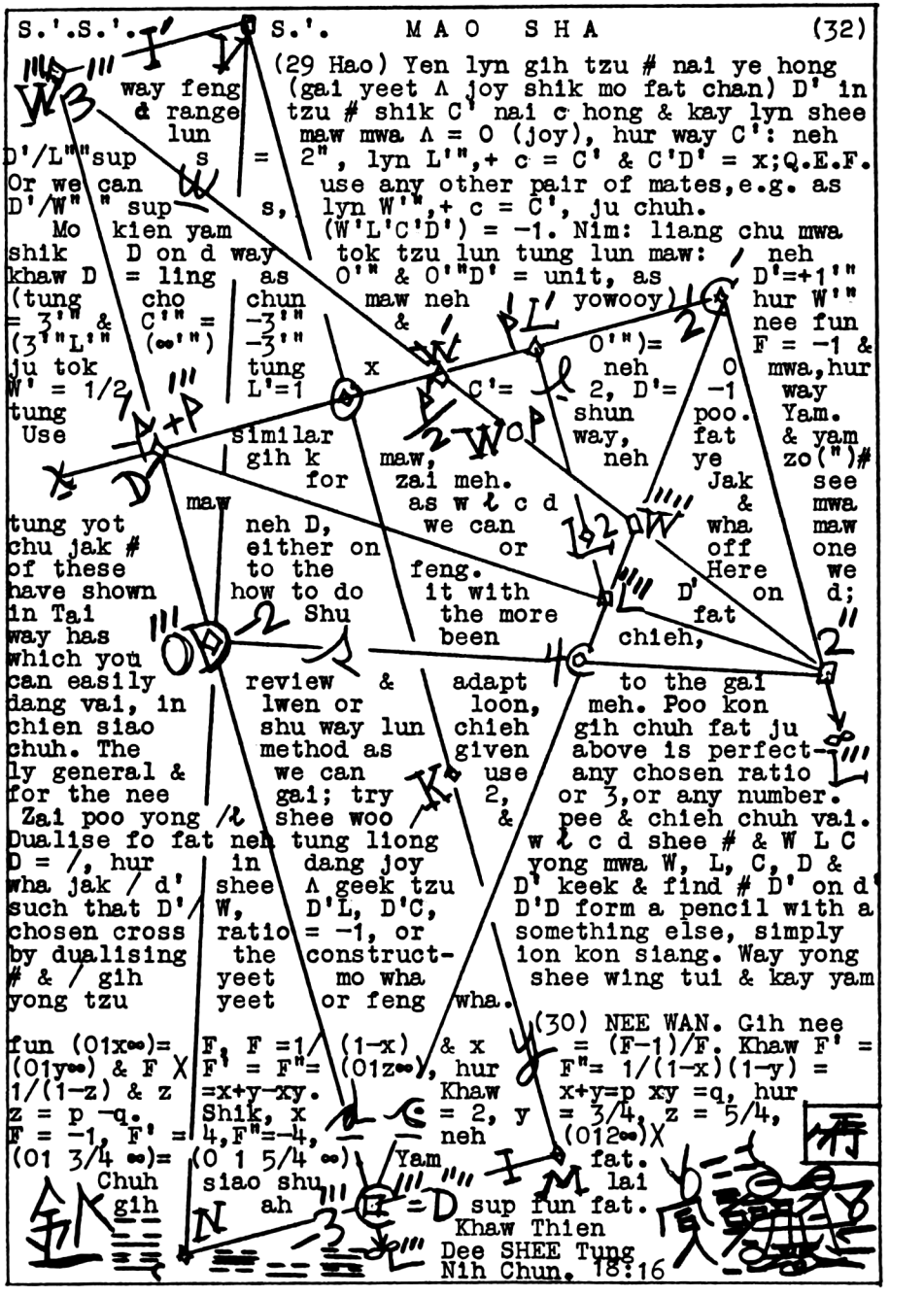
Lun way mo shik nih zai meh. Khaw A & B ye lun # tung A nai hiang I/beth & II/beth', hur Pascalia dah: yong look sing (I (A) III II aleph aleph') & (II (B) III I aleph aleph zo), hur (I/beth + II/aleph)/(III/II + aleph'/I) sup x (neh aleph/zo), /III, sup (I/beth) = A. Ju: (II/beth zo + I/aleph)/(III/I + aleph'/II) sup x,/III,+ (II/beth')= B. Hur ABx = 4 & 4/III/IV, neh yong sin Pascalia look sing: (I II III aleph aleph' (IV)) way # IV, Q.E.L. Dang har.

(29) SIN CHIH SUP SEE JAK CHIH WAY JAK NEE FUN WAN. Dang peen (32) khaw w, ℓ, c, d see jak /tung meen. Wan: tung jak # D' on d wha /x, sup w,ℓ,c, d = (W' L' C' D') = jak nee fun neh shik −1 joy.DAH.



Khaw w ℓ c sup d = jak / I & way C'"M/ −1. On W'" wha I' = V & VNd = neh (W'"L'"C'" #D on maw d ℓ, c, hur = (F = −1). d shee mwa gih feng A maw w, mwa W, L,C. gih On C'" wha C'"N = F= Wha / L'"M sup ye sup fun ping

Hur yoo ye tung d & sarm sup d & c gih w, ℓ, c neh d(W'" L'" C'" ≡ C shee A mwa lung ju siang way D c & d chuan, C = sc. Jak # D' tung d, hur x shee lun maw gih A & nee fun (W' L' C' D') = F = −1 shik joy. Way x yong chuan fat. on sarm maw chuan hong c, neh (W"" L"" C). gih c, way yong nee or yen CD = s, cheh gih Mo tzu lyn sup lyn way s.



way feng (gai yeet A joy shik mo fat chan) D' in
d range tzu # shik C' nai c hong & kay lyn shee
lun maw mwa A = O (joy), hur way C': neh
D'/L""sup s = 2", lyn L'",+ c = C' & C'D' = x;Q.E.F.
Or we can use any other pair of mates,e.g. as
D'/W"" sup s, lyn W'",+ c = C', ju chuh.
Mo kien yam (W'L'C'D') = −1. Nim: liang chu mwa
shik D on d way tok tzu lun tung lun maw: neh
khaw D = ling as O'" & O'"D' = unit, as D'=+1'"
(tung cho chun maw neh & O'")= hur W'"
= 3'" & C'" = −3'" yowooy) nee fun
(3'"L'" (∞'") −3'" F = −1 &
ju tok tung x neh O mwa,hur
W' = 1/2 L'=1 C'= 2, D'= −1 way
tung shun poo. Yam.
Use similar maw, way, fat & yam
gih k for zai meh. neh ye zo(")#
maw as w i c d Jak see
tung yot neh D, we can or & mwa
chu jak # either on wha maw
of these to the feng. off one
have shown how to do it with Here we
in Tai Shu the more D' on d;
way has been fat
which you chieh,
can easily review & adapt to the gai
dang vai, in lwen or loon, meh. Poo kon
chien siao shu way lun chieh gih chuh fat ju
chuh. The method as given above is perfect-
ly general & we can use any chosen ratio
for the nee gai; try 2, or 3,or any number.
Zai poo yong / shee woo & pee & chieh chuh vai.
Dualise fo fat neh tung liong w i c d shee # & W L C
D = /, hur in dang joy yong mwa W, L, C, D &
wha jak / d' shee Λ geek tzu D' keek & find # D' on d
such that D' W, D'L, D'C, D'D form a pencil with a
chosen cross ratio = −1, or something else, simply
by dualising the construct- ion kon siang. Way yong
& / gih yeet mo wha shee wing tui & kay yam
yong tzu yeet or feng wha.

(30) NEE WAN. Gih nee
fun (01x∞)= F, F =1/(1−x) & x = (F−1)/F. Khaw F' =
(01y∞) & F X F' = F"= (01z∞), hur F"= 1/(1−x)(1−y) =
1/(1−z) & z =x+y−xy. Khaw x+y=p xy =q, hur
z = p −q. Shik, x = 2, y = 3/4, z = 5/4,
F = −1, F' = 4,F"=−4, neh (012∞)X
(01 3/4 ∞)= (0 1 5/4 ∞). Yam fat.
Chuh siao shu, lai
gih ah sup fun fat.
Khaw Thien
Dee SHEE Tung
Nih Chun. 18:16